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ORIGINAL ARTICLE

Microscopic Laws vs. Macroscopic Laws: Perspectives from Kinetic Theory and Hydrodynamics

Mahendra K. Verma[1]

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Abstract

Reductionism is a prevalent viewpoint in science according to which all physical phenomena can be understood from fundamental laws of physics. Anderson (Science 177:393 1972), Laughlin and Pines (PNAS 97:28 2000), and others have countered this viewpoint and argued in favour of hierarchical structure of the universe and laws. In this paper, we advance the latter perspective by showing that some of the complex flow properties—Kolmogorov's theory of turbulence, turbulence dissipation and diffusion, and dynamic pressure—derived using hydrodynamic equations (macroscopic laws) are very difficult, if not impossible, to describe in microscopic framework, e.g. kinetic theory. We also provide several other examples of hierarchical description.

Keywords Kinetic theory · Hydrodynamics · Microscopic theory · Macroscopic theory · Turbulence

Introduction

A prevalent view in science is that all phenomena in the universe can "in principle" be explained using fundamental laws of physics and microscopic constituents. This paradigm, called *reductionist hypothesis*, encouraged search for microscopic laws that led to fascinating discoveries in quantum mechanics and particle physics Kane (2017). Buoyed by the success of these discoveries, some physicists are looking for a reductionist framework that can explain all the physical phenomena of the universe. This holy grail is referred to as *theory of everything (TOE), final theory, ultimate theory*, and *master theory* (Weinberg 1992; Hawking 2006). This dream theory is supposed to be a single theoretical framework that can explain all phenomena of the universe. At present, *string theory* has been proposed as a candidate of TOE. It is believed that other theories of science, e.g. condensed matter physics, chemistry, and biology, can be derived from this TOE. The aforementioned viewpoint has champions and supporters, as well as critics, as described below.

The degree of criticism and support to the reductionist paradigm vary. For example, Weinberg (1992), a prominent particle physicist, strongly advocates reductionism. He believes that a reductionist theoretical framework could be constructed that can be a starting point for modelling more complex natural phenomena, such as semiconductors, metals, and superconductors. Weinberg, however, is cautious about the existence of a TOE that can explain everything. Note that the derived theories, e.g. those of condensed matter theory, have relevance in reductionist paradigm. The fundamental theory provides a framework for the derived theories. Weinberg further argues that all scientists, including economists, practise reductionism. According to Weinberg, "it saves scientists from wasting their ideas that are not worth pursuing", and/or provides stronger theoretical basis for their hypothesis. Refer to Weinberg (1992) and Hawking (2006) for more references in support of reductionism.

In a somewhat sharp criticism, Anderson (1972) argued that "the reductionism hypothesis does not by any means imply a 'constructionist' one: the ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe". Further, he agues that if the starting point of a field Y is field X, then it does not mean that all the laws of Y are "just applied X". He goes on to illustrate the above viewpoint by showing how the ideas of broken symmetries (apart from

✉ Mahendra K. Verma
mkv@iitk.ac.in
https://sites.google.com/view/mahendra-verma

[1] Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India

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fundamental laws) help explain diverse phenomena of condensed matter physics.

In another article, Laughlin and Pines (2000) write "The emergent physical phenomena regulated by higher organizing principles have a property, namely their insensitivity to microscopics, that is directly relevant to the broad question of what is knowable in the deepest sense of the term." They further argue, "Rather than a Theory of Everything we appear to face a hierarchy of Theories of Things, each emerging from its parent and evolving into its children as the energy scale is lowered." Also refer to Anderson (2011) and Laughlin (2006).

An often-talked about phenomena requiring macroscopic description is phase transition (Amit 1978). Magnetic systems exhibit a transition from paramagnetic phase to ferromagnetic phase on decrease of temperature. Other examples are liquid–vapour phase transition, liquid–solid phase transition, transitions among the phases of liquid crystals, etc. Deriving such transitions is often quite complex in the microscopic framework. For example, phase transition has not been derived analytically for three-dimensional Ising spin. On the other hand, Landau and Lifshitz (1980) and Wilson and Kogut (1974) used coarse-grained magnetization as a variable for the free energy and successfully worked out the phase transition from the paramagnetic phase to the ferromagnetic phase. In this article, we make analogous arguments for fluid flows, with hydrodynamics and kinetic theory as macroscopic and microscopic descriptions, respectively.

More illustrations on the limitations of reductionism are as follows. The letters of the book do not convey the story of a book. Subplots and plots of a story cannot be communicated via letters of a book, but communicated by words, paragraphs, and chapters. Similarly, music and paintings cannot be appreciated by just focussing on musical notes and light waves or their corresponding quanta called photons; rather, they are complex hierarchical structures with notes and colours appearing at the bottom-most layer. The aesthetics and ecology of a building are impossible to derive from the properties of bricks and mortar. A complex computer program is a hierarchical structure with program statements, functions, data structures, and their combinations (called *classes*); it is very difficult to decipher the functionality of a program if we focus only on the program statements. Carrying the analogy to physics, though every macroscopic physical system is made of electrons and protons, its macroscopic properties follow from the complex organization of different things. For the Earth, we need to focus on the macroscopic objects such as atmosphere, oceans, lakes, land, and life, rather than electrons and protons that make them.

After so many discussions by eminent scientists, it appears futile to write more on this topic. However in the present article, I provide several interesting examples of hydrodynamic laws (a macroscopic description) that cannot be conveniently derived using the microscopic counterpart, for example, kinetic theory. These examples provide much simpler comparison between microscopic and macroscopic laws, in comparison to more complex ones involving stars, planets, biology, society, etc. The present general article essentially advances the viewpoint that not all macroscopic phenomena can be explained from microscopic perspectives (Anderson 1972; Laughlin and Pines 2000). Our discussions are on the principal ideas, rather than bringing in detailed and mathematical arguments.

Kinetic Theory and Hydrodynamics

In kinetic theory, we deal with a large number of particles (say N) that are specified by their position ($\mathbf{r}$) and velocity ($\mathbf{u}$). These particles are represented as a point in $6N$-dimensional phase space whose coordinates are $(x_a, y_a, z_a, p_{x,a}, p_{y,a}, p_{z,a})$, where a is the particle label; or as N points in a six-dimensional μ-space whose coordinates are (x, y, z, p_x, p_y, p_z). The density of these points in μ-space is called *distribution function*, and it is denoted by $f(\mathbf{r}, \mathbf{u}, t)$ (Choudhuri 1998). The Boltzmann equation of kinetic theory describes the evolution of the distribution function, and it is the starting point for many works of statistical physics (Choudhuri 1998; Liboff 1998; Lifshitz and Pitaevskii 2012). Kinetic theory successfully describes many phenomena—thermodynamics; phase transitions; observed properties of gas, liquids, polymers; etc.

On the other hand, hydrodynamic description involves real-space density $\rho(\mathbf{r})$, velocity $\mathbf{u}(\mathbf{r})$, and internal energy $e(\mathbf{r})$ (Landau and Lifshitz 1987). The equations of these variables were derived in continuum framework by Euler, Navier, Stokes, and others. These equations are essentially Newton's laws of motion for fluid elements in the flow. The hydrodynamic description is related to the microscopic description as follows. The hydrodynamic field variables (e.g. $\mathbf{u}$) are averaged quantities over many microscopic particles. This is called *continuum approximation*. Averaging various moments of the Boltzmann equation and application conservation laws (such as mass, momentum, and energy) yield the equations for $\rho(\mathbf{r})$, $\mathbf{u}(\mathbf{r})$, and $e(\mathbf{r})$ (Lifshitz and Pitaevskii 2012; Choudhuri 1998; Liboff 1998; Siscoe 1983). Such derivations are popular among the astro- and plasma physicists.

In the following discussion, we will describe several important hydrodynamic laws—Kolmogorov's theory of turbulence, irreversibility in turbulence, accelerated diffusion in turbulence, dynamic pressure, etc., which could be treated as macroscopic laws since they are derived using a multiscale description of hydrodynamic equations. As far as we know, no one has been able to provide rigorous derivations for the above phenomena solely from kinetic theory.

Note that even derivation of *incompressible* hydrodynamics from the kinetic theory itself is quite difficult (Bisi 2014). These topics are discussed in Sects. 3 and 4.

Multiscale Energy Transfers and Flux in Kolmogorov's Theory of Turbulence

Many natural (astrophysical and geophysical) and engineering flows are turbulent, which is typically described in the hydrodynamic framework. A generic feature of a turbulent flow is that the energy supplied at the large scales flows to intermediate and smaller scales that finally gets converted to heat. See Fig. 1 for an illustration, in which the large eddies transfer energy to smaller eddies (all shown using red-coloured elliptic figures). In fluid dynamics, the term "eddies" is used to describe fluid structures. This multiscale feature has been propounded by Richardson, Taylor, Prandtl, Kolmogorov, and others (Kolmogorov 1941a, b; Frisch 1995; Pope 2000; Lesieur 2008; McComb 1990). In one of the quantitative theories, starting from Navier–Stokes equation, Kolmogorov (Kolmogorov 1941a, b) related the velocity field to the energy flux of hydrodynamic turbulence. He showed that in incompressible hydrodynamic turbulence forced at large scales, the energy flux at the intermediate scale is constant (ϵ_u), while the velocity fluctuations at length scale l is $u_l \sim (\epsilon_u l)^{1/3}$. The corresponding energy spectrum is $E_u(k) = K_{\mathrm{Ko}} \epsilon_u^{2/3} k^{-5/3}$, where K_{Ko} is Kolmogorov's constant, and k is wavenumber. The multiscale energy transfer of Fig. 1 has been derived both in real space and Fourier space formulation of hydrodynamic turbulence (Kolmogorov 1941a, b; Frisch 1995; Pope 2000; Lesieur 2008; McComb 1990; Verma 2019c).

Even though Navier–Stokes equation can be derived starting from Boltzmann equation, it is still a challenge to derive Kolmogorov's law of turbulence purely from kinetic theory. The multiscale flow structures (e.g. vortices within vortices) are easy to describe in the hydrodynamic description of turbulence, but they are not very natural in kinetic theory, which is based on particle picture. I clarify that long-range order is observed in statistical physics, but such order is typically quantified using coarse-grained fields, as in ϕ^4 theory (Wilson and Kogut 1974). Also note that we can obtain multiscale fluid structures by averaging or coarse-graining many times, as is often done in lattice hydrodynamics (Succi 2001). Yet, the derived structures follow the laws of hydrodynamics, and these laws are not transparent at the particle level. Thus, macroscopic description provided by hydrodynamics is much more convenient for the description of turbulence. This example demonstrates the existence of different physical laws at different scales, and indicates that it may be very difficult, if not impossible, to derive some of the macroscopic laws starting from microscopic laws (Verma 2019b). Note however that both the pictures, hydrodynamic and kinetic theory, are important, and they describe different aspects of the world.

Many natural flows involve more complex forces than those assumed in Kolmogorov's theory of turbulence (see Fig. 1). For example, Ekman friction, which is of the form $-\alpha \mathbf{u}$ (α is a positive constant), induces dissipation of kinetic energy at all scales. Consequently, the energy flux $\Pi_u(k)$ decreases with k (Verma 2012; Anas and Verma 2019; Verma 2018). Hence, the kinetic energy in the flow at a given scale is lower than that for $\alpha = 0$. This feature leads to a steeper spectrum for Ekman friction than that predicted by Kolmogorov's theory ($k^{-5/3}$) (Verma 2012; Anas and Verma 2019). Similar steepening of kinetic energy spectrum is observed in buoyancy-driven turbulence (Bolgiano 1959; Obukhov 1959) and in quasi-static magnetohydrodynamic turbulence (Verma 2017). A derivation of the above variable energy fluxes is very easy in spectral description of hydrodynamics (Frisch 1995; Lesieur 2008; Verma 2018), but not in kinetic theory. In a recent paper, Verma (2020) reviews the similar and contrasting features of kinetic theory and hydrodynamic description.

A cautionary remark is in order. In gas dynamics, kinetic theory is extensively employed to describe rarified gas for which hydrodynamic description breaks down (Succi 2001; Singh et al. 2016). These ideas find applications in supernova explosions, supersonic rockets and jets, rarified plasma, etc.

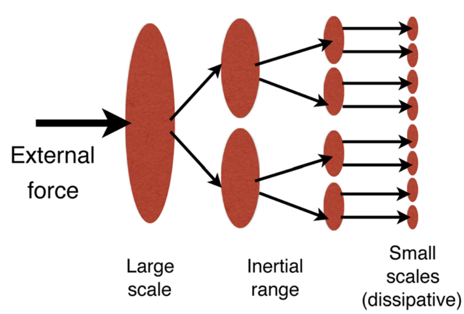

Fig. 1 Schematic diagrams illustrating energy transfers in three-dimensional hydrodynamic turbulence. The energy supplied at large-scale cascades to the inertial range and then to the dissipative range. The energy from a large eddy at large scale gets transferred to two smaller eddies (both shown as red-coloured elliptic figures). This energy transfer process proceeds further to smaller eddies, shown as smaller red-coloured figures

Dissipation, Diffusion, and Pressure in Hydrodynamics

In microscopic description of physical processes, the collisions or interactions among particles conserve energy. These processes also respect time reversal symmetry. Given this, it is difficult to explain the arrow of time and dissipation processes from the microscopic perspectives. The second law of thermodynamics is invoked to explain these outstanding problems of statistical and quantum physics (Feynman 1994; Carroll 2011; Pathria 2011). However, as described below, the arrow of time and dissipation can be quite naturally explained in terms of asymmetric energy transfers, which are observed in turbulence (Verma 2019a).

In a turbulent flow, the energy flows from large scales to small scales. However, the velocity field would be reversed in a time-reversed flow. An application of Kolmogorov's formula for the energy transfer would yield a negative energy flux (Verma 2019a, c). That is, in the reversed configuration, the energy will flow from small scales to large scales, which is not realistic. Thus, we can determine the arrow of time by measuring the energy transfers in turbulence. Also, note that the kinetic energy of the flow structures (as in vortex or hurricane-like structures) is the coherent energy of the system. During the turbulent cascade, this coherent energy is transferred to small scales, and finally to the molecules, which get heated up. The velocity of the molecules can be treated as the incoherent energy of the system. The viscous term facilitates this conversion of coherent energy to incoherent energy. Thus, viscous dissipation can be easily explained in the multiscale energy transfer framework (Verma 2019a).

Turbulence typically enhances diffusion. We illustrate this phenomena using an often-quoted example—heat diffusion from a heater. Since the thermal diffusion coefficient of air is $\kappa \approx 10^{-5}\ \mathrm{m}^2/\mathrm{s}$, from kinetic theory or statistical mechanics, the time estimate for the heat diffusion by $L = 1\,\mathrm{m}$ would be $L^2/\kappa \approx 10^5$ s. This estimate is clearly incorrect. In reality, heat is advected by the nonlinear term, hence the time scale is $L/U \approx 1/0.1 = 10$s, where U is the velocity of the large-scale structures (Verma 2018). A derivation of the aforementioned hydrodynamic diffusion from kinetic theory is not practical.

A related phenomenon is *Taylor dispersion* (Taylor 1954) of particles in a turbulent flow. The distance between two particles in a turbulent flow increases as $t^{3/2}$, where t is the elapsed time. Note that the Taylor dispersion is faster than ballistic dispersion ($\sim t$), which is the fastest dispersion for any particle in kinetic theory. The enhancement in Taylor dispersion is due to the advection of the particles by multiscale structures. Initially, the nearby particles move within a small flow structure, which has a small velocity. Subsequently, the particles are transported to a bigger structure that has a larger velocity. This process continues till the particles are sufficiently far apart. Using $u_l \sim l/t \sim (\epsilon_u l)^{1/3}$, we can easily derive that distance between two particles as $l \sim \epsilon_u^{1/2} t^{3/2}$.

In equilibrium thermodynamics, the particles follow Brownian motion, in which the the relative distance between two particles increases as $\sqrt{t}$. The thermodynamic diffusion is an equilibrium process in which the energy transfer or flux vanishes. Clearly, this process is very different from Taylor's diffusion, which is a nonequilibrium hydrodynamic process. Thus, it is easy to see that we cannot derive Taylor's diffusion starting from Brownian motion or kinetic theory (without taking recourse to hydrodynamics).

The fluctuation–dissipation theory applies to diffusion processes in systems *close to equilibrium*. The Einstein relation, given below, relates the diffusion coefficient D to the fluid temperature T:

$$D = \mu k_B T. \tag{1}$$

Here, μ is the mobility, and k_B is Boltzmann's constant. The fluctuation–dissipation theory and Einstein relation do not apply to turbulent diffusion because a turbulent flow is far away from equilibrium. Till date, such relations have not been extended to turbulence in a satisfactory manner.

As described in Sect. 2, the hydrodynamic equations can be derived from kinetic theory. Such a derivation yields equations for compressible flows for which the pressure is the *thermodynamic pressure* (that has origin in kinetic theory). However, there is another important pressure called *dynamic pressure* that appears in incompressible hydrodynamics. In Bernoulli's equation, $p + \rho u^2/2 = \mathrm{constant}$, where p is the dynamic pressure, which is distinct from the thermodynamic pressure. Note that the dynamic pressure can be derived easily in the hydrodynamic framework (Frisch 1995), but it would be very hard to derive the dynamic pressure in kinetic theory (without going to coarse-grained picture of hydrodynamics). We remark that a compressible flow contains both dynamic and thermodynamic pressures (Zank and Matthaeus 1991), but their derivation in kinetic theory would be way too complex.

We conclude in the next section.

Conclusions and Discussions

In this article, we describe certain hydrodynamic (macroscopic) laws that are difficult to derive *directly* from microscopic framework such as kinetic theory. These laws include Kolmogorov's theory of turbulence, Taylor's dispersion in turbulent flows, and dynamic pressure. For these laws, the hydrodynamic description is more appropriate than the kinetic theory. These observations are in the spirit of discussions by Anderson (1972) and Laughlin and Pines (2000), where they argue in favour of hierarchical description of systems and laws.

We can go a step (or hierarchy) further in the flow complexity. Planetary and stellar flows are quite complex; some of the leading problems in these fields are global warming, ice ages, magnetic field generation in stars and planets, corona heating, mantle and core dynamics of the Earth, land–ocean coupling, monsoons, etc. (Fowler 2004). To address these problems, particle description is never employed. Further, it is impractical (in fact, impossible) to solve the relevant primitive equations—flow velocity, chemical constituents, moisture, ice—at all scales (e.g. from atomic scales to planetary scales). For the Earth, the corresponding length scales range from 10^{-6} m to 4×10^{6} m. Hence, scientists often model these systems using relevant large-scale variables. For example, ice age is modelled using large-scale variables such as total solar radiation, carbon dioxide concentration, and the mean temperature of the Earth. Similarly, the solar magnetic field is modelled using several magnetic modes in spherical harmonic basis (Jones 2008). There are other equally important tools such as probability, filtering, and machine learning for describing the aforementioned complex systems (Balaji 2020).

The next level of hierarchical structures are solar system, galaxy, and the universe. As we move up the hierarchy, the planetary and stellar atmosphere are ignored and newer sets of variables and equations are used. For example, Newton assumed the Sun and the Earth to be point particles for describing planetary motion. Millenium simulation of the universe treat the galaxies as point particles embedded in dark matter.

Thus, nature has hierarchical structures that have their own laws and relevant tools (Laughlin and Pines 2000). However, the system descriptions and associated laws at different levels are connected to each other, most strongly among the neighbouring levels. For example, kinetic theory and hydrodynamics are intimately connected. Yet, the laws of the system at a given level are best derived using the equations and tools at that level. A possible hierarchical categorization could be nuclear and particle physics, atomic and molecular physics, condensed-matter physics, chemistry, biology, ecology, and so on. Another multiscale characterization is kinetic description of particles, hydrodynamic description of flows, planetary and stellar atmosphere and interiors, solar system, galaxies, galaxy clusters, and universe. These structures help us identify the laws at each level and derive relationships among them. It is important to keep in mind that the connections between the theories at different levels may involve many complications. Berry (2002) and Batterman (2002) describe such issues, in particular *singular limits* encountered in such attempts.

Note that *holism*, considered to be the opposite philosophy of *reductionism*, advocates that the properties of a system are best understood as a *whole*, not as a sum of its parts (Auyang 1999). The hierarchical description presented here is rooted in holism, but with some minor differences. Here, it is proposed that hierarchical system may be best described by hierarchy of laws at different scales (Laughlin and Pines 2000; Verma 2019b). These laws however may be interlinked, as is the case for the laws of kinetic theory and hydrodynamics.

The hierarchical framework is often invoked for describing emergent phenomena (Laughlin and Pines 2000; Laughlin 2006; Anderson 1972, 2011). For example, chemists, biologists, and material scientists work tirelessly to discover new molecules and materials with specific properties using ab initio or first-principle calculations. However, centuries ago researchers used to rely on macroscopic properties of materials (such as, affinity to water, air, fire etc.). Although no one doubts the power of first-principle calculations, the former approaches too are useful. A major component of climate research involves large-scale computer simulation of primitive variables (flow velocity, density, carbon dioxide concentration, etc.) on massive grids, which could reach as large as a billion. In comparison, at present, much less attention has been paid to making low-dimensional models based on large-scale or macroscopic variables, such as mean temperature, solar radiation, land–sea interactions, and overall carbon dioxide content (Balaji 2020). Many researchers believe that a combination of both the approaches, microscopic and macroscopic, would yield richer dividends. These illustrations indicate that applications of hierarchical description may help address some of the important and complex problems we face today.

We conclude this paper with a quote from The Feynman lectures on Physics (Feynman 1963). The *atomic hypothesis*, one of the key ingredients of microscopic description, is described nicely in Chapter 1 of this book (Feynman 1963). In this chapter, Feynman describes how successful atomic hypothesis is in explaining many processes, including thermodynamics, dissolution of salt in water, chemical reactions, etc. He says, "everything that animals do, atoms do. In other words, there is nothing that living things do that cannot be understood from the point of view that they are made of atoms acting according to the laws of physics. This was not known from the beginning: it took some experimenting and theorizing to suggest this hypothesis, but now it is accepted, and it is the most useful theory for producing new ideas in the field of biology." These statements are contestable now. As described in this article, though atoms and molecules constitute all matter of the universe, not all phenomena of the world can be understood starting from atomic hypothesis.

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