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Nonclassical viscosity and resistivity of the solar wind plasma

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Abstract. In this paper we have estimated typical viscosity and resistivity of the solar wind using turbulence phenomenology rather than *Branskii's* [1965] formalism. Since the solar wind is a collisionless plasma, in this paper we have assumed that the dissipation in the solar wind occurs at the proton gyroradius through wave-particle interactions. Using this dissipation length scale and the dissipation rates calculated using MHD turbulence phenomenology [*Verma et al.,* 1995], we estimate the viscosity and resistivity. The resisitivity has been recalculated using anomalous scattering time. We find that all of our transport quantities are several orders of magnitude higher that those calculated using classical theories of *Branginskii* [1965].

1. Introduction

The solar wind is a collisionless plasma; the distance traveled by the solar wind protons between two consecutive Coulomb collisions is approximately 3 AU [*Barnes,* 1979]. Therefore the dissipation in the solar wind involves wave-particle interactions rather than particle-particle collisions.. (For the observational evidence of the wave-particle interactions in the solar wind refer to the review articles by *Gurnett* [1991] and *Marsch* [1991, and references therein].) For these reasons, while calculating the transport coefficients in the solar wind, the scales of wave-particle interactions appear more appropriate than those of particle-particle interactions [*Braginskii*, 1965]. This is the main point of this paper.

Earlier *Montgomery* [1983] calculated the transport coefficients in the solar wind using *Braginskii*'s [1965] formalism, which is based on particle-particle collision. In this formalism the ion's kinematic viscosity ν_1 is

$$\nu_1 = \frac{3}{10}\frac{T_i}{\omega_i^2 \tau_i m_i}, \tag{1}$$

and the resistivity is

$$\lambda = \frac{m_e c^2}{4\pi n e^2 \tau_e}, \tag{2}$$

where T_i is the ion temperature, ω_i is the ion gyrofrequency, τ_i is the ion-ion collision time, m_i and m_e are the masses of the ions and electrons respectively, c is the speed of light, n is the number of particles per cm^3, and τ_e is the electron-proton collision time. For typical values of the solar wind parameters, *Montgomery* [1983] found both kinematic viscosity and resistivity to be of the order of 10^{-6} km^2 s^{-1}. Using the velocity of the large eddies as 20 km s^{-1} and length scale as 10^6 km, he obtained the Reynolds number to be of the order of 10^{13}. Note that *Montgomery* [1983] used ν_1 of *Braginskii* [1965] rather than ν_0. This is consistent with *Hollweg*'s [1985] result where he showed that the ν_0 terms are fully accounted for by the diagonal pressure tensor.

In fluid turbulence the dissipation length scale (Kolmogorov's microscale) l_d, kinematic viscosity, and dissipation rates are related by [*Lesieur*, 1989]

$$l_d \sim \left(\frac{\nu^3}{\epsilon}\right)^{1/4}. \tag{3}$$

The solar wind fluctuations have been observed to be turbulent, and their energy spectra follow approximately $k^{-5/3}$ power law [*Matthaeus and Goldstein*, 1982]. Motivated by this observation we attempt to estimate the viscosity and resistivity using turbulence phenomenology. The phenomenology of magneto-hydrodynamic (MHD) turbulence is not as developed as fluid turbulence phenomenology. At this moment it is not clear what would be the corresponding formula for viscosity in MHD turbulence. In this paper we have obtained a formula similar to equation (3) for MHD turbulence using simplified assumptions (see appendix).

For the solar wind, if we substitute in equation (3), a typical kinematic viscosity obtained by *Montgomery* [1983], and the dissipation rate obtained by *Tu* [1988] and *Verma et al.* [1995] (10^{-3} km^{2}s^{-3}), we find that $l_d \sim 10$ cm. This is much smaller than the proton gyroradius (100 km) where, according to the observations, transition from inertial range to dissipation range takes place. In this paper we argue that the dissipation length scale for the solar wind should be determined by the scale at which the wave-particle interaction occurs. We find that these estimates are consistent with the solar

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wind observations. Using this scale and the dissipation rate estimates, we obtain typical kinematic viscosity of the solar wind. We have also calculated resistivity from the scales obtained by turbulence phenomenologies.

In turbulence phenomenology it is argued that at the dissipation scale the smallest coherent fluid parcels disperse, that is, the fluid and the magnetic energies after this scale are zero. Here we refer to the dissipation scale as the transition scale between inertial and dissipation range. Also, note that the dissipation rate is the amount of energy supplied to the fluid at large scales; hence it can be determined by the large-scale forcing processes or from the energy spectra in the inertial range using the universal scaling laws. In work by *Verma et al.* [1995] the dissipation rates were determined from the energy spectra in the inertial range using the Kolmogorov-like MHD turbulence phenomenology [*Marsch,* 1990; *Matthaeus and Zhou,* 1989; *Zhou and Matthaeus,* 1990]. The turbulence phenomenologies also enable us to obtain transport coefficients as mentioned above. The advantage of these arguments is that the transport coefficients can be calculated without any reference to the dissipation mechanisms. (For discussion on dissipation processes in the solar wind refer to *Marsch et al.* [1982] and *Marsch*, [1990, and references therein].) The transport coefficients calculated here could find applications in modeling of solar wind evolution. Note, however, that the viscosity and resistivity calculated in this paper are to be used in fluid models of the solar wind.

Marsch et al. [1982] and *Gary* [1993] have calculated the damping rates of waves in space plasma using Vlasov theory (for further references, refer to reviews by *Marsch* [1991] and *Gurnett* [1991]). These calculations involve detailed nonthermal and highly non-Maxwellian distribution functions of particles and a variety of microinstabilities. The turbulent pulsation occurring as a result of plasma microinstabilities leads to an increased flux of particles and heat across the magnetic field that is confining the plasma; this phenomenon is referred to as anomalous diffusion and thermal conduction. R. Z. Sagdeev, unpublished notes, has shown that the anomalous diffusion coefficient due to microinstabilities is of the same order of magnitude as Bohm's diffusion. The viscosity and the resistivity calculated in this paper are not directly related to the transport because of microinstabilities, and hence they are not same as the anomalous diffusion coefficients referred to in the kinetic theory of fusion or space plasma. The viscosity and the resistivity calculated in this paper are basically fluid transport quantities. We use the scale of wave-particle interaction to estimate the length scale where the fluid and the magnetic energies disperse, as explained in the previous paragraph.

Note that the viscosity in a turbulent fluid is scale dependent [*Lesieur*, 1989]. The viscosity discussed in most of this paper is the one at the dissipation length scale, not at a large or intermediate length scale. The viscosity at large scale is called turbulent eddy viscosity; it is briefly discussed at the end of section 2. Note that the transport quantities in the solar wind vary with distance. In this paper we estimate these quantities at 1 AU.

In section 2 we will estimate the length scale at which wave-particle interactions take place. This will be the dissipation length scale for our calculation of the transport coefficients. Using the dissipation rates calculated earlier [*Tu*, 1988; *Verma et al.*, 1995] , we then estimate the kinematic viscosity and resistivity at the dissipation length scales. Toward the end of section 2, we also estimate the eddy viscosity of the solar wind. Section 3 contains conclusions.

2. Calculation

Verma et al. [1995] have calculated the dissipation rates in the solar wind streams using the Kolmogorov-like MHD turbulence phenomenology [*Marsch,* 1990; *Matthaeus and Zhou,* 1989; *Zhou and Matthaeus,* 1990]. The choice of this phenomenology over *Kraichnan*'s [1965] phenomenology or *Dobrowolny et al.*'s [1980] generalization of Kraichnan's phenomenology is motivated by the fact that the observed solar wind energy spectra tend to be closer to Kolmogorov's $k^{-5/3}$ power law than Kraichnan's $k^{-3/2}$ power law [*Matthaeus and Goldstein,* 1982]. Also, the temperature evolution study of *Verma et al.* [1995] shows that the predictions of the temperature evolution using the Kolmogorov-like model are in closer agreement with the observations than with those using *Kraichnan*'s [1965] or *Dobrowolny et al.*'s [1980] models. The reader is also referred to *Tu* [1988] for theoretical studies of turbulent heating in the solar wind.

The Kolmogorov-like phenomenology provides the energy spectra of fluctuations $\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}/\sqrt{4\pi\rho}$, where $\mathbf{u}$ is the velocity field fluctuation, $\mathbf{b}$ is the magnetic field fluctuation, and ρ is the density of the plasma. The quantities $\mathbf{z}^{\pm}$ represent the amplitudes of Alfvén waves having positive and negative velocity-magnetic field correlations, respectively. The energy spectra according to this phenomenology are

$$E^{\pm}(k) = C^{\pm} \left(\epsilon^{\pm}\right)^{4/3} \left(\epsilon^{\mp}\right)^{-2/3} k^{-5/3}, \qquad (4)$$

where $\epsilon^{\pm}$ are the dissipation rates of $\mathbf{z}^{\pm}$ fluctuations, and $C^{\pm}$ are Kolmogorov's constants for MHD turbulence. According to *Verma et al.* [1995], the dissipation rates per unit mass in the solar wind streams are of the order of 10^{-3} km^2 s^{-3}.

As mentioned in the introduction, we estimate the dissipation length scale from the theories of wave-particle interactions. The process by which Alfvén waves might be damped has been a subject of considerable research. It has been shown that the wave-particle resonance between MHD waves and ions occurs either in the form of the Doppler-shifted cyclotron resonance,

$$\omega - k_{\parallel} v_{\parallel} = n\Omega_i; (n = \pm 1, \pm 2, \cdots,) \qquad (5)$$

or in the form of the Landau resonance

$$\omega - k_{\parallel} v_{\parallel} = 0, \qquad (6)$$

where Ω_i is the cyclotron frequency of the ions, ω is the wave frequency, and $k_\parallel$ and $v_\parallel$ are the parallel components along the mean magnetic field of the wave number and the ion velocity vector, respectively [*Stix,* 1962; *Barnes,* 1979]. The question now arises whether the fluctuations in the solar wind can be damped by the above mechanisms.

For the solar wind, at 1 AU the cyclotron frequency Ω_i of the ions is of the order of 0.1 s^{-1}, and the thermal speed v is of the order of 50 km s^{-1} [*Barnes,* 1979]. Typical Alfvén speed $V_A = \omega/k$ at 1 AU is also 50 km s^{-1}. Also, note that the solar wind fluctuations are dominated by Alfvén waves; the compressive waves are damped at the early stages of its transit. Since $\omega/k \sim 50$ km/s $\sim v$, it appears that the Alfvén waves can be Landau damped. However, *Barnes and Suffolk* [1971] and *Barnes* [1979] argue against this. They show that the transverse Alfvén waves are exact solutions of the Vlasov-Maxwell equations for arbitrary amplitude, hence it cannot be damped. However, that is not correct either. It has been shown that all hydromagnetic waves, except the Alfvén mode with precise circular polarization, steepen and evolve into other modes or collisionless shocks [*Tidman and Krall,* 1971]. *Sagdeev and Galeev* [1969] showed that a linearly polarized Alfvén wave is unstable and that it decays to a backscattered Alfvén wave and magnetosonic waves. The magnetosonic waves thus generated get damped by Landau damping (see *Barnes* [1979, and references therein] for discussion on Landau damping of magnetosonic waves). *Hollweg* [1971] has obtained similar results. Hence the Alfvén waves in the solar wind can get damped by decaying to a magnetosonic waves which in turn gets damped by Landau damping.

Now the question arises, which waves in the solar wind are affected by the above process? The energy from the small and intermediate k (large wavelength) waves cascades to larger k waves due to nonlinear interaction arising from the $\mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm$ term of MHD equation [*Kraichnan,* 1965], and these waves do not get damped. At the dissipation scale the energy cascade stops. We conjecture that the decay of Alfvén waves to magnetosonic waves and the damping of the generated magnetosonic waves occur near the ion gyroradius $r = 100$ km. Therefore $k_d \sim 10^{-2}$ km^{-1}.

Regarding the cyclotron resonance, the small k Alfvén waves of the solar wind cannot be damped by this mechanism because $w \ll \Omega_i$ and $kv \ll \Omega_i$ when k is small (see equation (5)). Typical $k_{\min}$ in the solar wind is 10^{-7} km^{-1}, $\omega \sim kV_A$ is 10^{-5} s^{-1}, and $\Omega_i \sim 0.1$ s^{-1}. However, when k becomes large, it is possible for the waves to get damped by cyclotron damping. The approximate value of k where cyclotron resonance could occur is

$$k_d \sim \frac{\Omega_i}{V_A - V_\parallel} \sim \frac{0.1\ s^{-1}}{50\ \text{km/s}} \sim 2 \times 10^{-3}\ \text{km}^{-1}. \qquad (7)$$

Hence the dissipation length scale for the cyclotron damping is again of the order of 100 km. Therefore the dissipation wave number is $k_d \sim 10^{-2}$ km^{-1}. The solar wind observations show that at 1 AU the transition from inertial range to dissipation range occurs at around a length scale of 400 km (D. A. Roberts, private communication, 1995), a result consistent with our above arguments. In this paper we assume that the dissipation length scales for the fluid energy, the magnetic energy, and the energy of the Alfvén waves are all same (see appendix).

In the appendix we derive an expression for the viscosity ν in terms of dissipation rate ϵ and dissipation length scale (k_d^{-1}) to be

$$\nu \sim \lambda \sim \left(\frac{\epsilon}{k_d^4}\right)^{1/3}. \qquad (8)$$

Here we assumed that the fluid and magnetic energies are approximately equal and also that $E^+(k) \sim E^-(k)$. Under this condition, $\nu \sim \lambda$. We use this formula for our estimation of viscosity in the solar wind. Substitution of $\epsilon \sim 10^{-3}$ km^2 s^{-3} and $k_d \sim 10^{-2}$ km^{-1} in the above equation yields $\nu \sim 50$ km^2 s^{-1}. This result is very different from the one obtained by *Montgomery* [1983]. Note that the above viscosity is the ion viscosity. It is interesting to note that our estimate of the ion viscosity is close to Bohm's diffusion coefficient [*Chen,* 1974], which is

$$D_B = \frac{k_B T c}{16 e B} \sim 100\ \text{km}^2\text{s}^{-1} \qquad (9)$$

where k_B is the Boltzmann constant, T is the proton temperature, c is the speed of light, e is the electronic charge, and B is the mean magnetic field. It is not surprising that equation (8) with $\epsilon = 10^{-3}$ km^2 s^{-3} yields

$$\nu \sim \left(\frac{\epsilon k_B T c}{eB}\right)^{1/3} \frac{k_B T c}{eB} \sim \frac{k_B T c}{eB} \sim D_B \qquad (10)$$

because $\epsilon k_B Tc/eB \sim 1$. However, if $(\epsilon k_B Tc/eB)^{1/3}$ deviates far away from 1, then ν will have a dependence on ϵ; this feature of equation (8) is different from Bohm's diffusion formula. Note that Bohm's diffusion coefficient is called anomalous diffusion coefficient and is used in estimation of diffusion in fusion reactors.

The Reynolds number with $\nu = 50$ km^2 s^{-1}, the mean speed $U = 20$ km s^{-1}, and the length scale of 10^7 km is

$$Re = \frac{UL}{\nu} \sim 4 \times 10^6. \qquad (11)$$

The dissipation timescale is

$$\tau_d \sim \frac{1}{k_d v_d} \sim \frac{1}{(k_d^2 \varepsilon)^{1/3}} \sim 200\ \text{s} \qquad (12)$$

where v_d is the velocity at the dissipation scale. For the above expression we assumed that the Kolmogorov-like MHD turbulence phenomenology (equation (4)) is valid until $k = k_d$, therefore, $v_d \sim (E(k_d)k_d)^{1/2} \sim (k_d/\epsilon)^{1/3}$ ~ 2 km/s [*Lesieur,* 1989].

At this point we can compare *Montgomery*'s [1983] classical kinematic viscosity with our nonclassical viscosity. Montgomery's ν_1 is basically $0.3(2\pi)^2 r_p^2/\tau_{pp}$, where r_p is the proton gyroradius, and τ_{pp} is the proton-

proton collision time. Hence, typically, $\nu_1 \sim 10^{-6}$ km^2/s. However, using $\epsilon \sim v_d^3/l_d$ [*Lesieur*, 1989] and formula (8), we obtain $\nu \sim v_d l_d \sim 100$ km^2/s. The ratio of $\nu_{\mathrm{noncl}}/\nu_1 \sim (v_d \tau_{pp}/r_p)$, which is much bigger than 1.

The magnetofluid viscosity is dominated by ion viscosity. Therefore the viscosity estimated in the above discussion is primarily ion viscosity. The arguments in the appendix indicate that the resistivity may be of the same order as kinematic viscosity. Hence

$$\nu \sim \lambda \sim 50 \mathrm{km}^2/\mathrm{s} \tag{13}$$

Now we estimate the resistivity of the solar wind. The resistivity λ of magnetofluid is defined as [*Braginskii*, 1965]

$$\lambda = \frac{m_e c^2}{4\pi n e^2 \tau_e} \tag{14}$$

where m_e is the mass of the electron. For the solar wind typically $n \sim 5$ ions cm^{-3}. Using the electron-proton collision time for τ_e, *Montgomery* [1983] obtained $\lambda \sim 10^{-7}$ km^2 s^{-1}, which differs from the resistivity of equation (13) by 8 orders of magnitude. In the following discussion we obtain τ_e and subsequently λ from the scales of wave-particle interaction. *Schwartz et al.* [1981] showed that waves with frequencies near the ion gyro frequencies and wave vectors comparable with the inverse ion Larmor radii can provide a strong electron-wave coupling. There are some detailed calculations in this direction (for review and further references, refer to *Marsch* [1991]). We need to estimate what is the appropriate τ_e for equation (14). We would like to point out however that Coulomb collisions are important at least for the "core" electrons [*Scudder and Olbert*, 1979a, b], and their effect on temperature anisotropy, heat flux, etc., cannot be neglected. However, we are primarily interested in transport coefficients, and we will ignore Coulomb collisions as an approximation.

Priest [1982] argues that low-frequency ion-sound turbulence has an anomalous collision time, and this anomalous time scale should be used for anomalous conductivity. In this paper we are dealing with Alfvénic turbulence, and we estimate the anomalous scattering time in the following fashion: Eddy or kinematic viscosity can be interpreted as diffusion coefficients for coherent fluid parcels. The dissipation length scale discussed in this paper is the length scale where the smallest coherent fluid parcel disperse, that is, the fluid energy after this scale is zero (refer to Appendix). Similarly, the coherent magnetic structures are destroyed by resistivity at the dissipation scales. We assume in this paper that the dissipation length scales of both fluid and magnetic energy are k_d^{-1}. Therefore $l_d^e = k_d^{-1} \sim 100$ km. Since electrons are lighter particles, they move faster than protons; we assume that the relevant speed of the electrons at dissipation length scale is its thermal speed. Taking the electron temperature as 10^5 K, $v_d^e = 1000$ km/s. From these two scales, we can obtain the timescale that is $\tau_e \sim l_d^e/v_d^e \sim 0.1$ s. Our above arguments are in the same spirit as that of *Priest* [1982].

Now we can also estimate the resistivity by substituting the above anomalous scattering time in equation (14). Substitution of $\tau_e \sim 0.1$ s and $n = 5$ ions cm^{-3} yields $\lambda \sim 100$ km^2 s^{-1}. Note that the resistivity calculated here is close to the resistivity obtained in equation (13) using the dissipation rates. The magnetic Reynolds number will be

$$Re_M = \frac{UL}{\lambda} \sim 2 \times 10^6. \tag{15}$$

The solar wind magnetic Prandtl number, defined as ν/λ, appears to be of the order of unity. It is interesting to note that both renormalized viscosity $\nu(k)$ and resistivity $\lambda(k)$ are expected to scale as $\epsilon^{1/3}k^{2/3}$, where ϵ is the relevant dissipation rate, and k is the wave number [*Verma and Bhattacharjee*, 1995, and references therein]. Therefore the renormalized magnetic Prandtl number $\nu(k)/\lambda(k) \sim 1$. In this paper we are calculating $\lambda(k_d)$ and $\nu(k_d)$. It is reasonable to expect that Kolmogorov's 5/3 power law continues until $k = k_d$; therefore it is not surprising that our magnetic Prandtl number $\lambda(k_d)/\nu(k_d) \sim 1$. However, since the above numbers are only order of magnitude estimates, we can not make a definite prediction about the magnetic Prandtl number.

As mentioned in the Introduction, viscosity is scale dependent. The large-scale viscosity, called eddy viscosity, is $\nu_L \sim v_L L$, where L is the large-scale length and v_L is the large-scale fluctuating speed. Therefore, for the solar wind, $\nu_L \sim 20$ km/s $\times 10^8$ km$\sim 10^9$ km^2/s. This number is 7 orders of magnitude higher than the viscosity at dissipation length scale. These quantities could be useful for the study of solar wind evolution.

3. Conclusions

In this paper we have calculated the viscosity and the resistivity of the solar wind. Our calculation is based on turbulence phenomenologies. Here these transport coefficients have been estimated using the dissipation length scale and dissipation rates. The viscosity and the resistivity calculated in this paper are not directly related to the transport because of microinstabilities, hence they are not same as the anomalous diffusion coefficients referred to in the kinetic theory of fusion or space plasma. The viscosity and the resistivity calculated in this paper are basically fluid transport quantities. We have used the scales of wave-particle interactions just to estimate the length scale where the fluid and magnetic energies disperse. An important feature of this approach is that these calculations are independent of the dissipation mechanisms occurring in the dissipation range.

The solar wind plasma is collisionless. Therefore the wave-particle interactions become important while considering dissipation mechanisms in the wind. From the solar wind observations and from the rough estimates of scales where wave-particles interactions occur, we find that the dissipation length scale is close to the proton gyroradius (~ 100 km). In our calculation we also need turbulent dissipation rates occurring in the solar wind. In this paper we take the turbulent dissipation rates calculated by *Verma et al.* [1995a].

We find that a typical ion viscosity and resistivity is 50 km^2 s^{-1}. The corresponding Reynolds number (with ion viscosity) is around 10^6. The resistivity calculated by substituting anomalous scattering time in the resistivity formula is also around 200 km^2 s^{-1}. The magnetic Reynolds number is around 10^6. The magnetic Prandtl number is of the order of unity. The large-scale (eddy) viscosity of the wind is approximately 10^9 km^2/s.

All the transport quantities calculated by us are several orders of magnitude higher than those calculated earlier by *Montgomery* [1983] using the classical transport theory of *Braginskii* [1965]. Braginskii's formalism is based on particle-particle collision and is probably inapplicable for the solar wind plasma, which is collisionless. We believe that a formalism based on the wave-particle interaction should be applicable for the solar wind. However, we would like to point out that more work is needed for proper estimation of transport coefficients, for example, evaluation of $\nu_\pm$ when $E^+(k) \neq E^-(k)$ and $E^u(k) \neq E^b(k)$. Also, in our calculation the dissipation scales for all the energies $E^\pm(k)$, $E^u(k)$, and $E^b(k)$ have been assumed to be equal; this assumption needs closer examination through simulation and theory.

The transport coefficients presented in this paper are only rough estimates. However, we believe they could find applications in modeling of the solar wind and in numerical simulations.

Appendix

We derive an expression for viscosity in terms of energy dissipation rates and dissipation length scales. We use energy equations to derive this expression. The incompressible MHD equation in the absence of a mean magnetic field is [*Kraichnan*, 1965]

$$\frac{\partial}{\partial t}\mathbf{z}^\pm = -\mathbf{z}^\mp \cdot \nabla \mathbf{z}^\pm - \nabla p + \nu_+ \nabla^2 \mathbf{z}^\pm + \nu_- \nabla^2 \mathbf{z}^\mp \quad \text{(A1)}$$

$$\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b} \quad \text{(A2)}$$

$$\nu_\pm = \frac{1}{2}(\nu \pm \lambda) \quad \text{(A3)}$$

where $\mathbf{u}$ is the fluctuating velocity field, $\mathbf{b}$ is the fluctuating magnetic field in velocity units, p is the total pressure, ν is the kinematic viscosity, and λ is the resistivity. From this equation, under the assumption of isotropy of fluctuations, one can derive [*Orszag,* 1977]

$$\begin{array}{c} \frac{\partial}{\partial t} E^\pm(k) = -2\nu_+ k^2 E^\pm(k) \\ -2\nu_- k^2 \left[E^u(k) - E^b(k)\right] + T^\pm(k) \end{array} \quad \text{(A4)}$$

where $E^\pm(k)$ are the energy spectra of $\mathbf{z}^\pm$, $E^u(k)$ and $E^b(k)$ are the velocity and magnetic field energy spectra respectively, and $T^\pm(k)$ comes from the nonlinear term and involves triple correlations of $\mathbf{z}^\pm$. By integrating the above equation over the whole spectrum, we obtain

$$\begin{array}{c} \epsilon^\pm = -2\nu_+ \int_0^\infty k^2 E^\pm(k) dk \\ -2\nu_- \int_0^\infty k^2 \left[E^u(k) - E^b(k)\right] dk. \end{array} \quad \text{(A5)}$$

The term $T^\pm(k)$ upon integration over the whole spectrum yields zero [*Orszag*, 1977].

We make several assumptions to get an order of magnitude estimate of ν. We assume that the third term of equation (A5) vanishes. This condition will be satisfied either if $\nu_- = 0$ or $E^u(k) \sim E^b(k)$. Since the spectra $E^\pm(k)$ are usually strongly damped in the dissipation range, most of the contribution to the first integral of equation (A5) comes from k in the range of 0 to k_d. We also make a drastic assumption that $E^+(k) = E^-(k)$, and $\epsilon^+ = \epsilon^- = \epsilon$. These assumptions are justified only because we are making order of magnitude estimation of ν. To obtain precise values of $\nu_\pm$, we will have to analyze equation (A5) carefully. Now the substitution of $E^\pm(k)$ from equation (4) in equation (A5) yields

$$\nu \sim \lambda \sim \nu_+ \sim \left(\frac{\epsilon}{k_d^4}\right)^{1/3} \quad \text{(A6)}$$

Hence, given the dissipation rate ϵ and the dissipation length scale k_d^{-1}, we can estimate ν and λ.

In this derivation we have also assumed that the dissipation length scale for all the energies, that is, $E^\pm(k), E^u(k)$, and $E^b(k)$ are the same and are equal to k_d^{-1}.

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References

Barnes, A., Hydromagnetic waves and turbulence in the solar wind, in *Solar System Plasma Physics,* vol. *1,* edited by E. N. Parker, C. F. Kennel, and L. J. Lanzerotti, p. 249, North-Holland, New York, 1979.

Barnes, A., and G. C. J. Suffolk, Relativistic kinetic theory of the large-amplitude transverse Alfvén wave, *J. Plasma Phys., 5,* 315, 1971.

Braginskii, S. I., Transport processes in a plasma, *Rev. Plasma Phys.* (Engl. Trans.), *1,* 205, 1965.

Chen, F. F., *Introduction to Plasma Physics and Controlled Fusion,* Plenum, New York, 1974.

Dobrowolny, M., A. Mangeney, and P. Veltri, Fully developed anisotropic hydromagnetic turbulence in interplanetary space, *Phys. Rev. Lett., 45,* 144, 1980.

Gary, S. P., *Theory of Space Plasma Microinstability,* Cambridge Univ. Press, New York, 1993.

Gurnett, D. A., Waves and instabilities, in *Physics of the Inner Heliosphere II,* edited by R. Schwenn and E. Marsch, Springer-Verlag, New York, 1991.

Hollweg, J. V., Nonlinear Landau damping of Alfvén waves, *Phys. Rev. Lett., 27,* 1349, 1971.

Hollweg, J. V., Viscosity in a magnetized plasma: Physical interpretation, *J. Geophys. Res., 90,* 7620, 1985.

Kraichnan, R. K., Inertial-range spectrum of hydromagnetic turbulence, *Phys. Fluids, 8*, 1385, 1965.

Lesieur, M., *Turbulence in Fluids*, Martinus Nijhoff, Dordrecht, Netherland, 1989.

Marsch, E., Turbulence in the solar wind, in *Reviews in Modern Astronomy,* vol. 4, edited by G. Klare, p. 43, Springer-Verlag, New York, 1990.

Marsch, E., Kinetic physics of the solar wind plasma, in *Physics of the Inner Heliosphere II*, edited by R. Schwenn and E. Marsch, p. 45, Springer-Verlag, New York, 1991.

Marsch, E., C. K. Goertz, and K. Richter, Wave heating and acceleration of the solar wind ions by cyclotron resonance, *J. Geophys. Res., 87*, 5030, 1982.

Matthaeus, W. H., and M. L. Goldstein, Measurement of the rugged invariants of magnetohydrodynamic turbulence, *J. Geophys. Res., 87*, 6011, 1982.

Matthaeus, W. H., and Y. Zhou, Extended inertial range phenomenology of magnetohydrodynamic turbulence, *Phys. Plasmas, 1*(9), 1929, 1989.

Montgomery, D., Theory of hydromagnetic turbulence, in *Solar Wind Five,* NASA Conf. Publ., CP-2280, 107, 1983.

Orszag, S. A., Statistical theory of turbulence, in Les Houches Summer School of Theoretical Physics, edited by A. Balian and J. L. Peube, Gordon and Breach, New York, 1977.

Priest, E. R., *Solar Magnetohydrodynamics*, p. 79, D. Reidel, Norwell, Mass., 1982.

Sagdeev, R. Z., and A. A. Galeev, *Nonlinear Plama Theory,* Benjamin, White Plains, N. Y., 1969.

Schwartz, S. J., W. C. Feldman, an S. P. Gary, Wave-electron interactions in the high speed solar wind, *J. Geophys. Res., 86*, 4574, 1981.

Scudder, J. D., and S. Olbert, A theory of local and global processes which affect solar wind elcectrons, 1. The origin of typical 1 AU velocity distribution functions-Steady state theory, *J. Geophys. Res., 84*, 2755, 1979a.

Scudder, J. D., and S. Olbert, A theory of local and global processes which affect solar wind elcectrons, 2. Experimental support, *84*, *J. Geophys. Res.*, 6603, 1979b.

Stix, T. H., *Theory of Plasma Waves,* McGraw-Hill, New York, 1962,

Tidman, D. A., and N. A. Krall, *Shock Waves in Collisionless Plasmas,* Wiley-Interscience, New York, 1971.

Tu, C.-Y., The damping of interplanetary Alfvénic fluctuations and the heating of the solar wind, *J. Geophys. Res.*, *93*, 7, 1988.

Verma, M. K., D. A. Roberts, and M. L. Goldstein, Turbulent heating and temperature evolution in the solar wind plasma, *J. Geophys. Res., 100*, 19839, 1995.

Verma, M. K., and J. K. Bhattacharjee, Computation of Kolmogorov's constant in magnetohydrodynamic turbulence, *Europhys. Lett., 31,* 195, 1995.

Zhou, Y., and W. H. Matthaeus, Models of inertial range spectra of interplanetary magnetohydrodynamic turbulence, *J. Geophys. Res., 95*, 14,881, 1990.

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