

Role of turbulent dissipation and thermal convection in solar wind's temperature evolution

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Abstract. In this brief report we calculate the turbulent dissipation rates in the solar wind using the Kolmogorov-like MHD turbulence phenomenology with Kolmogorov's constants calculated by *Verma and Bhattacharjee* [1995]. We find that the turbulent heating cannot account for the total heating of the non-Alfvénic streams in the solar wind. Solar wind observations indicate that the thermal conduction could contribute significantly to the temperature evolution of the solar wind plasma. In this paper we theoretically estimate the contribution of turbulent thermal convection to the temperature evolution; the theoretical estimates are consistent with observations.

Introduction

The proton temperature T of the solar wind, regarded here as a single magnetofluid parameter, is observed to decrease slower than adiabatic cooling which yields $T(r) \propto r^{-4/3}$, where r is the distance from the sun. [Schwenn, 1983; Marsch *et al.*, 1983; Schwartz and Marsch, 1983; Gazis, 1984; Freeman and Lopez, 1985; Lopez and Freeman, 1986; Freeman *et al.*, 1992]. The observed $T(r)$ is proportional to $r^{-0.7}$ in the outer heliosphere and is proportional to $r^{-0.9}$ in the inner heliosphere, with a variation of approximately ± 0.1 in the exponent depending on velocity and other factors. These observations indicate that the protons in the solar wind are heated in transit. Note that in this paper we will not discuss the temperature evolution of the electrons and the alpha particles in the solar wind. Also, the dissipation rates in the solar wind vary with distance from the Sun; here we estimate typical dissipation rates in the solar wind (at 1 AU). The solar wind has spatial inhomogeneities which play an important role in its evolution [Zhou and Matthaeus, 1990b]. We assume local homogeneity and isotropy in the wave number space so that turbulence phenomenology can be applied to the solar wind [Zhou and Matthaeus, 1990b; Tu *et al.*, 1984; Tu, 1988]. The temperature evolution under these assumptions appears reasonable [Tu *et al.*, 1984; Tu, 1987, 1988; Verma *et al.*, 1995].

Various attempts have been made to explain the observed temperature evolution and heating in the solar wind. The proposed sources of heating are shocks [Whang *et al.*, 1990], turbulence [Tu *et al.*, 1984; Tu, 1987, 1988; Verma *et al.*, 1995], interactions with neutral particles [Isenberg *et al.*, 1985] etc. Heat conduc-

tion can lead to a redistribution of heat and can have an important contribution to the temperature evolution of the solar wind [Gazis, 1984]. Recently, Verma *et al.* [1995] have studied the temperature evolution of the protons in the solar wind plasma and have estimated the dissipation rates which would be sufficient to heat the plasma to the observed level. They invoked turbulence as a heating mechanism in the solar wind. However, they used a free parameter in their calculation. Recently, Verma and Bhattacharjee [1995] have theoretically calculated this parameter for non-Alfvénic streams (described below). We estimate a typical turbulent dissipation rate in the solar wind using the constants estimated by Verma and Bhattacharjee [1995]. We also estimate the contribution of turbulent thermal convection to temperature evolution of the solar wind plasma.

Tu *et al.* [1984], Tu [1988], and Verma *et al.* [1995] (henceforth referred to as paper I) calculated turbulent heating in the solar wind using the existing MHD turbulence phenomenologies, Kolmogorov-like MHD turbulence phenomenology [Marsch, 1990; Matthaeus and Zhou, 1989; Zhou and Matthaeus, 1990a] and Dobrowolny *et al.*'s generalized Kraichnan phenomenology [Dobrowolny *et al.*, 1980; Kraichnan, 1965]. In the Kolmogorov-like phenomenology [Marsch, 1990; Matthaeus and Zhou, 1989; Zhou and Matthaeus, 1990a], the energy spectra $E^\pm(k)$ of fluctuations $\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}$ (\mathbf{u} is the velocity fluctuation, and \mathbf{b} is the magnetic fluctuation in velocity units) are

$$E^\pm(k) = C^\pm (\epsilon^\pm)^{4/3} (\epsilon^\mp)^{-2/3} k^{-5/3} \quad (1)$$

where ϵ^\pm are the dissipation rates of \mathbf{z}^\pm , and C^\pm are Kolmogorov's constants for MHD turbulence. The fluctuations \mathbf{z}^\pm are Alfvénic fluctuations with positive and negative velocity and magnetic field correlations. Note that using this scheme, we can obtain the dissipation rates using the energy spectra of the wind and the con-

stants C^\pm ; for these estimates we do not need any detailed knowledge of the dissipation mechanism. In paper I it was found that the normalized cross helicity $\sigma_c = 2\mathbf{v} \cdot \mathbf{b} / (v^2 + b^2)$, a measure of velocity and magnetic field correlation, plays an important role in determining dissipation in the solar wind. The constants of the phenomenologies were treated as free parameters, and it was assumed that $C^+ = C^- = C$. When we choose $C = 1.0$ for nonAlfvénic streams and $C = 8.0$ for Alfvénic streams, turbulent heating is sufficient to heat the plasma to the observed level (paper I).

Recently, *Verma and Bhattacharjee* [1995] calculated the values of the constants C^\pm using direct interaction approximation (DIA) of *Kraichnan* [1959]. They found that the C^\pm are not universal constants as is the case in fluid turbulence, but that they depend on the Alfvén ratio r_A (ratio of kinetic and magnetic energy) and the normalized cross helicity σ_c . So far, *Verma and Bhattacharjee* [1995] have succeeded in obtaining the constants when $\sigma_c = 0$. In Table 1 we list some of the values of these constants. These constants are in good agreement with the simulation results [*Verma et al.*, 1996]. In this brief report we will use the constants C obtained by *Verma and Bhattacharjee* [1995].

Tu et al. [1984] have calculated the dissipation rates in the solar wind using *Kraichnan's* MHD turbulence phenomenology [*Kraichnan*, 1965] (also see *Dobrowolny et al.* [1980]), in which the fluid and magnetic energy spectra are given by

$$E^b(k) \sim E^u(k) = A(\epsilon B_0)^{1/2} k^{-3/2} \quad (2)$$

where E^u and E^b are the fluid and magnetic energies respectively, A is *Kraichnan's* constant, and B_0 is the mean magnetic field or the magnetic field of the largest eddies. *Kraichnan's* constant A is believed to be of the order of 1. We emphasize the Kolmogorov-like phenomenology because its predictions appear most consistent with the observed energy spectra of $-5/3$ for both z^\pm in the solar wind [*Matthaeus and Goldstein*, 1982; *Marsch and Tu*, 1990].

Thermal conduction can redistribute heat in the solar wind and can have a significant effect in the temperature evolution of the solar wind. *Hundhausen* [1972], *Gazis* [1984], and *Marsch et al.* [1983] performed observational studies on the thermal conduction of protons and electrons; they found that thermal conduction can modify the temperature significantly from the adiabatic evolution. *Hundhausen* [1972] calculated the heat

flux using classical transport coefficients of *Braginskii* [1965]. However, the solar wind plasma is collisionless and turbulent; therefore the thermal conductivity calculated using classical transport coefficients may be inappropriate. It is commonly observed that turbulence enhances the heat flux due to turbulent convection by large eddies [*Landau and Lifshitz*, 1987]. The heat flux due to turbulence usually dominates the thermal conduction by electrons. Motivated by this phenomenon, we argue that a similar enhancement could occur in the solar wind. We derive two alternative formulas for turbulent thermal diffusivity, one using turbulent eddy convection, and the other by a modification of *Hollweg's* formula for anomalous thermal conduction [*Hollweg*, 1975].

In this paper we estimate the heat flux due to turbulent eddy convection, which is a fluid phenomenon. Therefore we will ignore the kinetic effects which are important for heat transport by particles at microscopic levels. The kinetic effects must be important for electron heat flux, but for protons, turbulent convection probably dominates the heat transport. The other assumption regarding heat flux is decoupling of electron and proton heat fluxes. For this reason we assume that the temperature of the protons in the solar wind is affected by the heat flux of protons alone, not by the heat flux of electrons. The decoupling of electron and proton heat fluxes has also been argued in the classical formalism of *Braginskii* [1965]; it hinges on the fact that the proton-electron collision frequency is very small as compared to electron-electron or proton-proton collision frequencies [see *Hundhausen*, 1972; *Priest*, 1982].

Shocks are one of the potential heat sources for the solar wind. *Whang et al.* [1990] have performed observational and simulational studies on the corotating shocks in the solar wind between 1 and 15 AU, and they have shown that in this region, shocks are a major heating source in the solar wind, with most of the heating confined to 1-5 AU. They found that the increase in the entropy per shock is, on average, 0.8×10^{-23} J/K/proton. They found approximately 400 shocks in the 1-15 AU region which yield a dissipation rate ϵ_{shock} of approximately 10^{-3} km²/s³. They have also derived that the temperature variation due to the observed entropy increase yields $T(r) \propto r^{-0.53}$.

Results

In paper I we derived a formula for the temperature evolution of the solar wind protons due to turbulent heating. Other sources of heating were not included in that paper. However, one can easily generalize the equation for the temperature evolution when the effects of shocks and heat conduction are also included [see *Priest*, 1982]. The generalized equation is

$$\frac{dT}{dr} + \frac{2R}{C_V} \frac{T}{r} = \frac{\Sigma}{UC_V} \left(\epsilon_{\text{turb}} + \epsilon_{\text{shock}} + \frac{\nabla \cdot \mathbf{q}}{\rho} + Q \right), \quad (3)$$

where T is temperature, r is radial distance from the

Table 1. Kolmogorov's Constants for MHD Turbulence for Various Values of Alfvén Ratio and Normalized Cross Helicity

r_A	σ_c	C^+	C^-
0.5	0.0	4.04	4.04
1.0	0.0	3.38	3.38
2.0	0.0	2.58	2.58
5.0	0.0	1.92	1.92

Data from *Verma and Bhattacharjee* [1995].

Sun, R is Rydberg's constant, C_V is specific heat per unit mass at constant volume, Σ is mass per unit mole (≈ 1 g for the solar wind plasma), U is solar wind mean speed, ϵ_{turb} is the turbulent dissipation rate per unit mass, ϵ_{shock} is the dissipation rate per unit mass due to shocks, \mathbf{q} is heat flux, ρ is density, and Q is the sum of contributions from other sources. Here, as well as in paper I, we have assumed a constant speed for the solar wind, hence $\rho \propto r^{-2}$. We make this assumption to simplify the analysis even though the density variation is observed to differ marginally from r^{-2} .

In paper I the best fit to the observed temperature evolution of the solar wind was obtained when $C = 1.0$ for non-Alfvénic wind and $C = 8.0$ for Alfvénic wind. The dissipation rate at 1 AU was of the order of $10^{-3} \text{ km}^2/\text{s}^3$. In the outer heliosphere, most of the solar wind streams have been observed to be non-Alfvénic ($\sigma_e \sim 0$) with $r_A \sim 0.5$ (in the inertial range, i.e., $E^u(k)/E^b(k)$). From Table 1, $C = 4.04$ [Verma and Bhattacharjee, 1995] for these streams. Therefore the correct turbulent dissipation rate is approximately (refer to (1))

$$\epsilon_{\text{turb}} = \epsilon_{\text{paper I}} \left(\frac{C_{\text{paper I}}}{C} \right)^{3/2} \quad (4)$$

$$\sim \frac{10^{-3}}{4^{3/2}} \sim 1.25 \times 10^{-4} \text{ km}^2 \text{ s}^{-3}.$$

Note that in paper I it was found that $\epsilon_{\text{paper I}}$ is the dissipation rate required to heat the solar wind plasma to the observed temperature. From the above expression $\epsilon_{\text{turb}} < \epsilon_{\text{paper I}}$ in the outer heliosphere. Therefore turbulent heating is not sufficient to heat the plasma to the observed temperature. The remaining contribution to the temperature evolution can be provided by shocks, thermal conduction, and other mechanisms. Whang *et al.* [1990] showed that shock heating is significant and is of the order of $10^{-3} \text{ km}^2/\text{s}^3$. In the latter part of this section we will discuss the observational and theoretical estimates of the contribution of thermal conduction and turbulent convection to the temperature evolution of the solar wind.

In the inner heliosphere, the fluctuations are somewhat fluid dominated. Therefore, C in the inner heliosphere is smaller than C of the outer heliosphere (see Table 1). If we take the inertial range r_A in the range of 1 to 2, C will be in the range of 3.38 to 2.58 (see Table 1). With these constants the turbulent heating contribution in the inner heliosphere is higher than in the outer heliosphere. Still it is at most $C^{-3/2} \sim 25 - 30\%$ (approximately one-quarter) of the of observed heating (heating rate of paper I) in the solar wind. The corotating shocks are absent in the inner heliosphere. Therefore, shock heating is presumably not significant for non-Alfvénic streams in the inner heliosphere. Hence, neither shocks nor turbulence can provide enough heating in the inner heliosphere. It is possible that thermal conduction plays an important role.

Regarding Alfvénic streams, in paper I it was found that the observed temperature evolution of the Alfvénic streams are in good agreement with the predictions of Kolmogorov-like phenomenology if Kolmogorov's con-

stant for MHD turbulence C is equal to 8.0. Currently, we do not have good estimates of the constants C^\pm for Alfvénic streams; future calculations of C^\pm will give us a better handle on the turbulent dissipation rates of Alfvénic streams in the solar wind.

Kolmogorov's constant for Alfvénic streams being close to 8.0 appears unrealistic. If Kolmogorov's constant is lower than 8.0, then the predicted dissipation rate by Kolmogorov-like turbulence phenomenology will be higher than what has been estimated in paper I (note that paper I sets an upper limit on the dissipation rate). This appears to be a contradiction. One possible resolution to this contradiction is that Kolmogorov-like turbulence phenomenology is not valid for Alfvénic streams, possibly because turbulence in the Alfvénic streams may not have reached steady state. This has been conjectured by Marsch [1991] and Grappin *et al.* [1991]. The nonlinear interactions may be distributing energy among large and intermediate wavenumbers, with only a small amount of energy flowing into the dissipation range. Therefore, the real dissipation rate could be lower than that predicted by the phenomenology, which assumes that the system is in steady state. Any modeling of "approach toward steady state turbulence" will shed light on the evolution of energy spectra and help us to correctly estimate turbulent heating in the Alfvénic streams of the solar wind.

Having seen that the turbulent heating is probably not sufficient to heat the non-Alfvénic streams to the observed temperature, we now consider the contribution of thermal conduction ($\nabla \cdot \mathbf{q}/\rho$ term of (3)) to the temperature evolution of the solar wind.

Hundhausen [1972], Gazis [1984], Marsch *et al.* [1983], Marsch and Richter [1984] have performed observational studies on thermal conduction in the solar wind. According to Hundhausen [1972], the heat fluxes $\mathbf{q}_{i,e}$, by protons and electrons, are 10^{-5} and $0.007 \text{ ergs cm}^{-2} \text{ s}^{-1}$ respectively. Gazis [1984] found that the conduction rate is $(2.5 \pm 1.0) \times 10^{-2} \text{ ergs cm}^{-2} \text{ s}^{-1}$; this heat flux is the sum of the electron and the proton heat fluxes because Gazis used a one-fluid equation. Marsch and Richter [1984] estimated that the proton heat flux \mathbf{q}_i is speed dependent and is approximately $10^{-4} \text{ ergs/cm}^2 \text{ s}$.

In this paper we will only concentrate on the proton heat flux. For the electron heat flux the kinetic effects probably play an important role. However, for the proton flux the turbulent convection probably dominates the microscopic kinetic effects. For this reason, for the following discussion we assume that the temperature of the protons in the solar wind is affected by the heat flux of the protons alone, not by the heat flux of the electrons. For classical transport calculations, it is widely believed that the coupling between electrons and protons is weak because the proton-electron collision frequency is very small as compared to the electron-electron or the proton-proton collision frequencies. This argument is not well founded because of low collisionality in the solar wind; still, in the absence of a well-formulated theory, we make the above assumption.

In the following discussion we will estimate the heat flux using classical transport coefficients, the eddy transport coefficient, and *Hollweg's* [1975] formula. According to the classical formalism of *Braginskii* [1965], the parallel and perpendicular heat conductivity K (of $\mathbf{q} = K\nabla T$) are

$$K_{\parallel} \sim \rho C_V v l,$$

and

$$K_{\perp} \sim \frac{\rho C_V v l}{(\omega_i \tau_i)^2}$$

where v is the thermal speed of the protons, l is the mean proton-proton collision length, ω_i is the proton gyrofrequency, and τ_i is the proton-proton collision time. Substitution of typical values of the solar wind parameters at 1 AU, $v \sim 50$ km/s, $l \sim 10^{13}$ cm, and $\omega_i \tau_i \sim 10^6$ yields $K_{\parallel} \sim 5 \times 10^4$ ergs/cm s K, and $K_{\perp} \sim 5 \times 10^{-8}$ erg/cm s K. These constants give $q_i \sim KT/r \sim 5 \times 10^{-4}$ erg/cm² s, with most of the conduction dominated by parallel conductivity. These results are in the order of magnitude range of the observed values of *Hundhausen* [1972], *Marsch et al.* [1983], *Marsch and Richter* [1984], and *Gazis* [1984]. However, since the solar wind plasma is collisionless and turbulent, application of classical transport formulas appears questionable. Thus, in the following discussion, we attempt to estimate thermal diffusivity due to turbulence.

In fluid turbulence it is observed that turbulence enhances thermal diffusion. This is because the large eddies carry heat more effectively than the electrons at a microscopic level. In fluids the turbulent thermal diffusivity due to convection K_{turb} can be estimated by $\rho C_V v l / Pr$, where v and l are the large-scale velocity and length scales respectively [*Landau and Lifshitz*, 1987], and Pr is the Prandtl number. Since the solar wind is a turbulent plasma, we apply this formula for the solar wind as well. MHD turbulence phenomenologies are not as sound as Kolmogorov's fluid turbulence phenomenology; however, we estimate K using the existing turbulence phenomenology as a first approximation. We assume that Pr is of the order of 1 for the solar wind. With $v \sim 20$ km/s (speed of large eddies) and $l \sim 10^{13}$ cm (size of the large eddies), $K_{\text{turb}} \sim 2 \times 10^4$ erg/cm s K. This result, however, is close to the classical K_{\parallel} . From turbulent K_{turb} the heat flux q_i again will be of the order of 10^{-4} erg/cm² s. Note that the large-scale eddies are not affected by the microscopic kinetic process, therefore, for our estimates of K_{turb} we do not need to have a detailed understanding of the dissipation processes occurring at the microscopic level.

As an alternative to the classical heat conductivity, *Hollweg* [1975] proposed that the anomalous electron heat flux q_e should be expressed as

$$q_e \cong (3/2)n_e K T_e v_{\text{crit}} \alpha \hat{\mathbf{b}} \quad (5)$$

where v_{crit} is a critical speed associated with the instability criterion for the heat conduction driven wave, $\hat{\mathbf{b}}$ is a unit vector along the mean magnetic field \mathbf{B}_0 with

the radial component pointing outward from the Sun, and α is a bugger factor that *Hollweg* estimates to lie in the range 2.0 – 7.0. The v_{crit} has been estimated to be of the order of the proton thermal speed (50 km/s). For typical values of the solar wind parameter, these arguments yield $q_e \sim 10^{-4}$ erg/cm² s. This is lower than the observational estimates ($q_e \sim 0.02$ erg/cm² s) and from what is estimated by classical transport formula.

We conjecture that the proton heat flux q_i due to turbulent convection could also be expressed using a formula similar to (5),

$$q_i \cong (3/2)n_i K T_i v_{\text{crit}} \quad (6)$$

In the above we argued that heat flux in a turbulent fluid or plasma is primarily dominated by turbulent convection, which depends on large length scales and large velocity scales. Therefore we take the large-scale speed, where the large-scale instabilities (e.g., shear) occur, as v_{crit} . With typical values of solar wind parameters ($v_{\text{crit}} \sim 20$ km/s) we obtain $q_i \sim 10^{-4}$ erg/cm² s, which is in the same range as observed in the solar wind. It must be emphasized that these statements are mere conjectures with plausible arguments offered in their favor. These arguments must be tested using detailed simulations and observations. However, these estimates appear to yield results that are close to the observed results and are encouraging.

Now we can estimate the contribution of heat flux to the temperature evolution and compare it with those by turbulent dissipation and shock heating. To this end we construct a quantity

$$H_i = -\frac{\nabla \cdot \mathbf{q}_i}{\rho} \sim \frac{q_i}{r\rho}. \quad (7)$$

At 1 AU with $q_i \sim 10^{-4}$ erg/cm² s, we obtain $H_i \sim 10^{-4}$ km² s⁻³. Our order of magnitude estimates are in general agreement with the observational results of *Marsch and Richter* [1984], *Marsch et al.* [1983], and *Hundhausen* [1972]. The contribution of turbulent convection to the temperature evolution H_i appears to be 1 to 2 orders of magnitude lower than turbulent dissipation and shocks; still it may be comparable and could have significant effects. In the inner heliosphere, H_i due to turbulent thermal convection will be somewhat higher because in the inner heliosphere the temperature is higher and r is lower than in the outer heliosphere. Therefore turbulent thermal conduction could potentially affect the temperature evolution in the inner heliosphere significantly. Note that the corotating shocks are absent in the inner heliosphere; therefore shock heating contribution is presumably not significant for non-Alfvénic streams in the inner heliosphere. Of course, other sources, e.g., stream-stream interactions and neutral ions, may also provide significant heating in the inner heliosphere.

As mentioned earlier, *Gazis* [1984] estimated the thermal energy flux at 1 AU in the solar wind and found it to be equal to $(2.5 \pm 1.0) \times 10^{-2}$ erg/cm² s, which corresponds to $\nabla \cdot \mathbf{q}/\rho \sim 2 \times 10^{-2}$ km² s⁻³, that is

approximately 10 times larger than the dissipation rate estimated in paper I. However, note that in Gazis' formalism, the thermal condition was considered the only source of entropy increase [see Gazis, 1984, equation (24)]; therefore the heating rate calculated by Gazis was an over-estimate. This is not surprising, since Gazis [1984] sets an upper limit on the total heat flux $\mathbf{q} (q_i + q_e)$.

Regarding thermal conduction, it must be noted that all three theoretical estimates, classical, turbulent, as well as that of Hollweg [1975], yield ion heat fluxes close to what is observed. Here we have emphasized the turbulent convection and Hollweg's argument because classical arguments may not be applicable to the collisionless and turbulent solar wind plasma. However, at this stage the turbulent conduction and Hollweg's arguments are conjectures, and they need further examination.

Conclusions

We have estimated the contributions of turbulent dissipation to the heating of the solar wind using Kolmogorov's constants for MHD turbulence calculated theoretically by Verma and Bhattacharjee [1995]. We find that for the non-Alfvénic streams, turbulent heating contributes only partly to the total heating in the solar wind. The remaining contribution to the temperature evolution should be provided by the corotating shocks, turbulent thermal convection, stream-stream interactions, interactions with the neutral ions, and other sources. The corotating shocks are present in the outer heliosphere. As shown by Whang *et al.* [1990], shocks could be one of the major heating sources in the outer heliosphere, at least in 1-15 AU where they have been studied, and could provide a major fraction of the heating. In the inner heliosphere the corotating shocks are absent; therefore turbulent dissipation and turbulent thermal convection may provide significant contributions to the temperature evolution.

For the Alfvénic streams, so far we do not have theoretical values of Kolmogorov's constants C^\pm . Therefore proper estimation of turbulent dissipation rates for Alfvénic streams is lacking. However, Kolmogorov's constant 8.0 used in paper I appears unrealistic. It is probably somewhat lower. With a lower constant, Kolmogorov-like turbulence phenomenology will predict dissipation rates higher than the upper limit calculated in paper I. This contradiction appears to reinforce the conjecture by Marsch [1991] and Grappin *et al.* [1991] that the Alfvénic streams have not reached steady state, and the energy is just being distributed among various modes, with only a fraction of the energy flowing into the dissipation range and heating the plasma. Future investigations on the "approach to steady state" will help us to properly estimate turbulent heating in Alfvénic streams and to understand other related problems.

Hundhausen [1972], Gazis [1984], and Marsch *et al.* [1983] performed observational studies of heat fluxes in solar heating. Hundhausen [1972] estimated classi-

cal thermal conductivity using one- and two-fluid models. Since the solar wind plasma is turbulent and collisionless, we estimate turbulent thermal convection in the solar wind using a formula used in fluid turbulence; this formula is based on turbulence phenomenology. We have also obtained another formula for turbulent thermal diffusivity by modifying Hollweg's [1975] expression for anomalous electron heat flux. Both diffusivities, turbulent heat diffusivity due to convection, and Hollweg's modified formula yield significant $\nabla \cdot \mathbf{q}/\rho$, which are in the order of magnitude range of turbulent dissipation and shock heating rates. However, we would like to point out that the turbulent thermal convection and Hollweg's arguments are mere conjectures.

The order of magnitude calculations show that the contribution of thermal convection to the solar wind temperature evolution is significant both in the inner as well as outer heliosphere. In the absence of corotating shocks in the inner heliosphere, turbulent thermal convection probably plays an important role in the temperature evolution there.

The estimates calculated in this paper shed some light on the contributions of various sources to the temperature evolution of the solar wind. A proper modeling of the sources will be useful in detailed studies of temperature and momentum evolution of the solar wind.

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References

- Braginskii, S. I., Transport processes in a plasma, *Rev. Plasma Phys.*, **1**, 205, 1965.
- Dobrowolny, M., A. Mangeney, and P. Veltri, Fully developed anisotropic hydromagnetic turbulence in interplanetary space, *Phys. Rev. Lett.*, **45**, 144, 1980.
- Freeman, J. W., and R. E. Lopez, The cold solar wind, *J. Geophys. Res.*, **90**, 9885, 1985.
- Freeman, J. W., T. Totten, and S. Arya, A determination of the polytropic index of the free streaming solar wind using improved temperature and density radial power-law indices (abstract), *Eos, Trans. AGU*, **73**, 238, 1992.
- Gazis, P. R., Observation of plasma bulk parameters and the energy balance of the solar wind between 1 and 10 AU, *J. Geophys. Res.*, **89**, 775, 1984.
- Grappin, R., M. Velli, and A. Mangeney, "Alfvénic" versus "standard" turbulence in the solar wind, *Ann. Geophys.*, **9**, 416, 1991.
- Hollweg, J. V., Waves and Instabilities in the solar wind, *Rev. Geophys.*, **13**, 263, 1975.
- Hundhausen, A. J., *Coronal Expansion and Solar Wind*, Springer-Verlag, New York, 1972.
- Isenberg, P. A., P. P. Chih, and L. A. Fisk, The heating of solar wind by interplanetary neutral gas, *J. Geophys. Res.*, **90**, 12,040, 1985.
- Kraichnan, R. H., The structure of isotropic turbulence

- at very high Reynolds numbers, *J. Fluid Mech.*, **5**, 497, 1959.
- Kraichnan, R. H., Inertial-range spectrum of hydromagnetic turbulence, *Phys. Fluids*, **8**, 1385, 1965.
- Landau, L. D., and E. M. Lifshitz, *Fluid Mech.*, p. 211, Pergamon, New York, 1987.
- Lopez, R. E., and J. W. Freeman, Solar wind proton temperature-velocity relationship, *J. Geophys. Res.*, **91**, 1701, 1986.
- Marsch, E., Turbulence in the solar wind, in *Reviews in Modern Astronomy*, Vol. 4, edited by G. Klare, p. 159, Springer-Verlag, New York, 1990.
- Marsch, E., MHD turbulence in the solar wind, in *Physics of the Inner Heliosphere II*, edited by R. Schwenn and E. Marsch, p. 159, Springer-Verlag, New York, 1991.
- Marsch, E., and A. K. Richter, Helios observational constraints on solar wind expansion, *J. Geophys. Res.*, **89**, 5386, 1984.
- Marsch, E., and C. Y. Tu, On the radial evolution of MHD turbulence in the inner heliosphere, *J. Geophys. Res.*, **95**, 8211, 1990.
- Marsch, E., K.-H. Mühlhäuser, H. Rosenbaur, and R. Schwenn, On equation of state of solar wind ions derived from Helios measurements, *J. Geophys. Res.*, **88**, 2982, 1983.
- Matthaeus, W. H., and M. L. Goldstein, Measurement of rugged invariants of magnetohydrodynamic turbulence, *J. Geophys. Res.*, **87**, 6011, 1982.
- Matthaeus, W. H., and Y. Zhou, Extended inertial range phenomenology of magnetohydrodynamic turbulence, *Phys. Fluids B*, **1** (9), 1929, 1989.
- Priest, E. R., *Solar Magnetohydrodynamics*, D. Reidel, Norwell, Mass., 1982.
- Schwartz, S. J., and E. Marsch, The radial evolution of a single solar wind parcel, *J. Geophys. Res.*, **86**, 4574, 1981.
- Schwenn, R., The "average" solar wind in the inner heliosphere: Structure and slow variations, in *Solar Wind Five*, NASA Conf. Publ., CP2280, 485, 1983.
- Tu, C.-Y., Explanation for the radial variation of the temperature of protons within the trailing edge of high speed streams between 1 and 5 AU, in *Solar Wind Six*, edited by V. J. Pizzo, T. Holzer, and D. G. Sime, Tech. Note NCAR/TN-306 + Proc, 593, 1987.
- Tu, C.-Y., The damping of interplanetary Alfvénic fluctuations and the heating of the solar wind, *J. Geophys. Res.*, **93**, 7, 1988.
- Tu, C.-Y., Z. Pu, and F. Wei, The power spectrum of interplanetary Alfvénic fluctuations: Derivation of the governing equations and its solutions, *J. Geophys. Res.*, **89**, 9695, 1984.
- Verma, M. K., and J. K. Bhattacharjee, Computation of Kolmogorov's constant in magnetohydrodynamic turbulence, *Europhys. Lett.*, **31**, 195, 1995b.
- Verma, M. K., D. A. Roberts, and M. L. Goldstein, Turbulent heating and temperature evolution in the solar wind plasma, *J. Geophys. Res.*, **100**, 19839, 1995a.
- Verma, M. K., D. A. Roberts, M. L. Goldstein, S. Ghosh, and W. T. Stribling, A numerical study of the nonlinear cascade of energy in MHD turbulence, *J. Geophys. Res.*, in press, 1996.
- Wang, Y. C., S. Liu, and L. F. Burlaga, Shock heating of the solar wind plasma, *J. Geophys. Res.*, **95**, 18,769, 1990.
- Zhou, Y., and W. H. Matthaeus, Models of inertial range spectra of interplanetary magnetohydrodynamic turbulence, *J. Geophys. Res.*, **95**, 14881, 1990a.
- Zhou, Y., and W. H. Matthaeus, Transport and turbulence modeling of solar wind fluctuations, *J. Geophys. Res.*, **95**, 10,291, 1990b.

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