Comment on "On two-dimensional magnetohydrodynamic turbulence" [Phys. Plasmas 8, 3282 (2001)]

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Biskamp and Schwarz [Phys. Plasmas **8**, 3282 (2001)] have reported that the energy spectrum of two-dimensional magnetohydrodynamic turbulence is proportional to $k^{-3/2}$, which is a prediction of Iroshnikov–Kraichnan phenomenology. In this Comment some earlier results are reported which conclusively show that for two-dimensional magnetohydrodynamic turbulence, Kolmogorov-like phenomenology (spectral index 5/3) is a better model than Iroshnikov–Kraichnan phenomenology; these results are based on energy flux analysis.
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In a recent paper, Biskamp and Schwarz[1] discuss energy spectrum and structure functions for two-dimensional (2-D) magnetohydrodynamic (MHD) turbulence. Based on numerical calculation of the energy spectrum, they claim that the spectrum of 2-D MHD turbulence agrees with Iroshnikov–Kraichnan's (IK) $k^{-3/2}$ law with some modifications. The purpose of the present comment is to show that the above claim is inconclusive. We bring to notice here an alternative point of view based on energy cascade rates which supports Kolmogorov's energy spectrum $K^{-5/3}$ for 2D as well as three-dimensional (3-D) MHD turbulence; this result is reported in Verma *et al.*[2] and Dar.[3]

MHD turbulence phenomenologies are discussed in Verma *et al.*[2] For zero velocity-magnetic correlation $(\mathbf{u}\cdot\mathbf{b})$, IK phenomenology predicts

$$E^u(k)=E^b(k)=A(\Pi V_{\mathrm{A}})^{1/2}k^{-3/2}, \tag{1}$$

where Π is the total energy flux, V_{A} is the Alfvén velocity, and A is a universal constant. Dobrowolny *et al.*[4] have generalized IK's arguments for nonzero $\mathbf{u}\cdot\mathbf{b}$ and showed that the energy cascade rates $\Pi^{\pm}$ of $\mathbf{z}^{\pm}=\mathbf{u}\pm\mathbf{b}$ are equal irrespective of the E^+ and E^- ratio, i.e.,

$$\Pi^+=\Pi^-\propto\frac{1}{B_0}E^+(k)E^-(k)k^3. \tag{2}$$

Marsch[5] proposed Kolmogorov-like phenomenology in which

$$E^{\pm}(k)=K^{\pm}\frac{(\Pi^{\pm})^{4/3}}{(\Pi^{\mp})^{2/3}}k^{-5/3}. \tag{3}$$

This is also a limiting case of a generalized phenomenology of Matthaeus and Zhou.[6] Clearly,

$$\frac{E^-(k)}{E^+(k)}=\frac{K^-}{K^+}\left(\frac{\Pi^-}{\Pi^+}\right)^2. \tag{4}$$

Biskamp and Welter,[7] Verma *et al.*,[2] Dar,[3] and Biskamp and Schwarz[1] have numerically computed the spectral exponents for 2-D MHD turbulence. Biskamp and Welter[7] support IK's $k^{-3/2}$ energy spectra, but Verma *et al.*[2] and Dar[3] find numerical uncertainties too significant to be able to distinguish between the exponents 3/2 and 5/3 (see Fig. 1 of Ref. 2). Biskamp and Schwarz[1] do not provide the error bars for the spectral indices (see Fig. 7 of Ref. 1). Since 5/3 and 3/2 are so close, the claims of Biskamp and Schwarz in favor of 3/2 may not be conclusive. As stated by them, the intermittency exponents do not clarify the matter any further. On the other hand, based on energy flux studies, Verma *et al.*[2] and Dar[3] could show quite conclusively that Kolmogorov-like phenomenology models 2-D MHD turbulence better than IK phenomenology.

Verma *et al.*[2] and Dar[3] numerically computed the energy fluxes Π^+ and Π^- for various E^-/E^+ ratios. The cascade rates of majority species (larger of E^- and E^+) were always found to be greater than those of the majority species. To illustrate we have plotted $\Pi^{\pm}$ for $E^-/E^+\approx 0.2$ in Fig. 1 (taken from Dar[3]). The same results are observed in Verma *et al.*,[2] however, the error bars in Dar[3] are relatively smaller ($\approx$5%) because of better averaging. Clearly $\Pi^+>\Pi^-$. The incongruence of the above result with the IK predictions [Eq. (2)] clearly indicated that IK phenomenology is not valid for 2-D MHD turbulence. For E^-/E^+ in the range of 0.2 to 1, Verma *et al.*[2] and Dar[3] find that

$$\frac{E^-}{E^+}\approx\left(\frac{\Pi^-}{\Pi^+}\right)^2. \tag{5}$$

This result is in agreement with the predictions of Kolmogorov-like phenomenology for MHD turbulence with $K^+=K^-$ [Eq. (4)]. Intermittency corrections to energy fluxes

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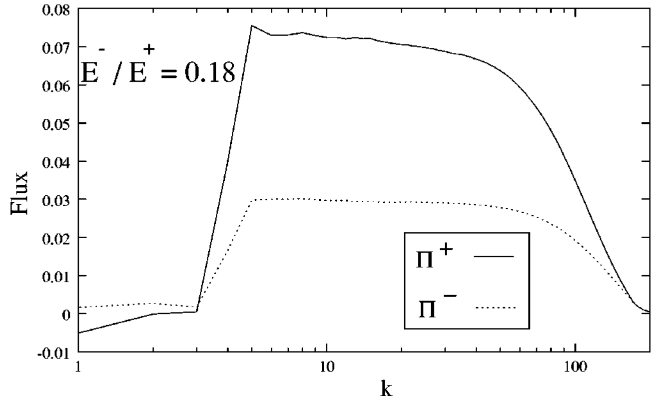

FIG. 1. Energy fluxes $\Pi^\pm$ vs k for a two-dimensional run with $B_0=0.0$ and $E^-/E^+=0.18$. Here $\Pi^+>\Pi^-$. The figure is taken from Dar (Ref. 3).

are typically small. Hence, we do not expect the large difference in energy fluxes to result from intermittency. This way Verma *et al.*[2] and Dar[3] showed that for 2-D MHD turbulence, Kolmogorov-like phenomenology is a better model than IK phenomenology. Recent advances on theoretical fronts also tend to indicate that Eq. (4) may even be valid for smaller E^-/E^+ with $K^+\neq K^-$.

Current theoretical and numerical papers[8,9] argue that Kolmogorov's energy spectrum in MHD is due to local Alfvén effects. The Alfvén waves are scattered by the "local mean magnetic field," rather than the global mean magnetic field. Hence, the effective time scale will be comparable to the nonlocal time scale resulting in Kolmogorov's energy spectrum for MHD turbulence. The above argument is expected to hold in both 2-D and 3-D.

To conclude, Verma *et al.*[2] and Dar's[3] results based on energy fluxes support Kolmogorov's spectrum for 2-D MHD turbulence. We believe Biskamp and Schwarz's claim favoring $k^{-3/2}$ energy spectrum is incorrect.

Note added in proof: The authors thank Professor Biskamp for pointing out the paper of Grappin *et al.*[10] However, for high normalized cross helicity ($\approx$0.9), Verma *et al.*[2] and Dar[3] find the exponents $m^\pm$ of $E^\pm\approx k^{-m^\pm}$ to be in the range of 1.5–1.7, but the ratio $\Pi^+/\Pi^-\gg 1$ (5 to 10). Hence numerical results of Verma *et al.*[2] and Dar[3] are not in agreement with the predictions of Grappin *et al.*[10] that $\Pi^+/\Pi^-\approx m^+/m^-$.

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