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Asymmetric energy transfers in driven nonequilibrium systems and arrow of time

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Abstract. Fundamental interactions are either fully or nearly symmetric under time reversal. But macroscopic phenomena may have a definite arrow of time. From the perspectives of statistical physics, the direction of time is towards increasing entropy. In this paper, we provide another perspective on the arrow of time. In driven-dissipative nonequilibrium systems forced at large scale, the energy typically flows from large scales to dissipative scales. This generic and multiscale process breaks time reversal symmetry and principle of detailed balance, thus can yield an arrow of time. In this paper we propose that conversion of large-scale coherence to small-scales decoherence could be treated as a dissipation mechanism for generic physical systems. We illustrate the above processes using turbulence as an example. In the paper we also describe exceptions to the above scenario, mainly systems exhibiting no energy cascade or inverse energy cascade.

1 Introduction

Fundametal forces – gravity, electrodynamics, and strong nuclear – exhibit time reversal symmetry. Weak nuclear force however exhibits a small violation of this symmetry. Hence, from the perspectives of fundamental interactions, forward and backward motion of a physical system are indistinguishable apart from a small violation for weak nuclear force [1–3]. Thus, in dynamics, the forward and time-reversed trajectories of particles are equally probable (except small deviation for weak interactions). However, such behaviour is not observed for macroscopic systems, as we describe below.

Typical physical and biological systems have a definite arrow of time. For example, stars, planets, and living beings take birth, live, and then die. A mixture of oxygen and nitrogen molecules evolves towards a uniform distribution of the molecules, but uniformly-distributed set of molecules do not separate into different species. That is, systems typically evolve from order (less entropy) to disorder (more entropy). This phenomena is explained using the second law of thermodynamics according to which the entropy of an isolated system cannot decrease [1–7]. Boltzmann's H-theorem forms a basis for the above law. Note however that Boltzmann's H-theorem itself assumes *molecular chaos hypothesis*, according to which the velocities of colliding particles are uncorrelated, and are independent of position of the particles [7,8].

Some researchers attribute the arrow of time in chaotic systems to *sensitivity to initial condition* [9]. In a chaotic system, most initial conditions take the system to chaotic configurations with higher entropy, and only a small set of initial conditions evolve to ordered states. The other mechanisms invoked for explaining arrow of time are measurements in quantum systems [10], chaos induced decoherence [11], etc. In the present paper we present another perspective from energy transfers in nonequilibrium systems.

Some of the leading examples of driven-dissipative nonequilibrium systems are turbulence, earthquakes, crack propagation, fragmentation, free market economy, astrophysical flows, etc. Several common features among them are (a) energy supply at large scales; (b) energy cascade from large scale to intermediate scale and then to small scale; (c) dissipation at small scales; and (d) multiscale physics with energy transfers across scales [4,5,12]. Each of the above systems are covered in vast literature. Here, we take turbulence as an illustrative example because it has been widely studied, and it is somewhat more familiar to physicists [12–16]. Note that the entropy of the above systems increases due to the dissipation, hence the present picture is consistent with the second law of thermodynamics. Yet, as we show in this paper, energy transfers provide further insights into the direction of time in nonequilibrium systems.

Unidirectional energy flux (from large scales to small scales) breaks the time reversal symmetry in turbulence. The nonlinear energy transfer is proportional to the triple product of the velocity Fourier modes. Hence, a change in the sign of the velocity field due to time reversal will change the sign of the energy flux. Therefore, in a time-reversed version of a turbulent flow, the energy will

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flow from small scales to large scales, contrary to real systems. Thus, the sign of the energy flux provides another indicator for the arrow of time – in three-dimensional (3D) hydrodynamics, positive energy flux implies forward in time, while negative energy flux would imply backward in time. We detail these issues in Section 2 of this paper.

It is important to define the time-reversal operation. For particle dynamics, the time reversal operation is $\mathbf{u} \to -\mathbf{u}$ and $t \to -t$, where $\mathbf{u}$ is the velocity of the particle at time t. This operation preserves time reversal symmetry in Newton's laws, $m\dot{\mathbf{u}} = \mathbf{F}$, if $\mathbf{F}$ is symmetric under time reversal. As described in the first paragraph, the fundamental forces, except weal nuclear force, are symmetric under time-reversal operation. The corresponding transformation for the velocity field of Navier-Stokes equation is $\mathbf{u}(t,\mathbf{r}) = -\mathbf{u}(-t,\mathbf{r})$ and $t \to -t$. Navier-Stokes equation without viscosity (also called Euler equation) is symmetric under time reversal, but the viscous term breaks the time reversal symmetry. This is consistent with the fact that friction breaks the timer reversal symmetry [1–3,17].

Note however that an external magnetic field, externally-imposed handedness, and chirality [18–20] break the time reversal symmetry. However, if all the constituents of the system including the currents that generates the magnetic field or chirality are reversed, then the system would preserve time reversal symmetry.

Davidson [21], Xu et al. [22], Jucha et al. [23], and others studied time irreversibility in turbulence. Davidson [21] attributes the arrow of time in a turbulent flow to chaotic advection arising due to the nonlinear terms. Using experimental data, Xu et al. [22] studied the fluctuations in power injection, p, to a Lagrangian particle moving in a turbulent flow. They showed that the probability distribution of p is asymmetric under $p \to -p$ transformation, and thus breaks the time reversal symmetry. Cencini et al. [24] observed similar behaviour in their direct numerical simulation and in a shell model; for a shell model, the velocity of the Lagrangian particle was taken to be $\sum \Re[u_n]$, where u_n is the velocity variable. Jucha et al. [23] quantified the irreversibility in turbulence using the relative motion between two particles. They showed that the difference between the forward and backward dispersion of the two particles varies as t^3, where t is the time elapsed. Piretto et al. [25] studied irreversibility in two-dimensional enstrophy cascade using a shell model.

The aforementioned past works on irreversibility in turbulence are primarily based on properties of Lagrangian particles embedded in the flow. The present paper however is focussed on the energy transfers in Eulerian framework, or in a snapshot of a turbulent flow. We demonstrate how energy transfers and flux provide important clues on the arrow of time; the energy transfers break the principle of detailed balance due to asymmetric energy transfers from large scales to small scales. We also show that the above scenario is applicable to many other nonequilibrium systems. in addition, we argue that the dissipation of kinetic energy at small scales and its subsequent conversion to heat offers an interesting recipe for introducing dissipation in a generic multiscale system. Towards the end of the paper we argue how the ideas of energy transfers could possibly be extended to the universe and many-body quantum systems.

The outline of the paper is as follows: In Section 2, we describe how asymmetric energy transfers in turbulence provide important inputs on the direction of time. In Section 3 we describe energy transfer issues for some of the nonequilibrium systems other than turbulence. In Section 4 we argue that the conversion of large-scale coherent energy to small-scale incoherent energy could be treated as a generic dissipation mechanism in nonequilibrium systems. Section 5 contains discussion on turbulent systems exhibiting inverse or zero energy cascade. We conclude in Section 6.

2 Direction of time in turbulent systems

In this section we describe the essential physics of three-dimensional (3D) hydrodynamic turbulence, and then the arrow of time in such flows. Consider an incompressible hydrodynamic flow that is forced at large length scales (L_f, forcing scale). According to the celebrated Kolmogorov's theory [12–16,26], in a turbulent flow, the energy supplied at large scales cascades to intermediate scales (called *inertial range*) and then to dissipative scales. See Figure 1a for an illustration. Note that the energy supply rate, the energy flux in the inertial range, and the energy dissipation rate are all equal (denoted by ϵ_u).

In the above macroscopic picture, which is based on *continuum approximation*, the velocity field at a position $\mathbf{r}$ is averaged over many microscopic particles around $\mathbf{r}$. This description of the velocity field is sensible up to Kolmogorov length scale η, which is approximately equal to $(\nu/\epsilon_u^3)^{1/4}$, where ν is the kinematic viscosity of the fluid. The microscopic physics, e.g. kinetic theory, describes the dynamics of particles at length scales smaller than η because the particles at these scales are in quasi-equilibrium. However, in the inertial range ($l > \eta$), the system is dynamic and in a nonequlibrium state (see Fig. 1a). Thus, the length scale η plays a major role in separating the coherent hydrodynamic structures with random motion at microscopic scales. We illustrate these features using an example. When we pour milk in a coffee cup, the large fluid structures of milk successively breaks into a hierarchy of structures, until it diffuses at microscales where the macroscopic coherent energy is destroyed as disorder or heat.

Kolmogorov [13,14] showed that the energy flux ϵ_u is related to the structure function:

$$\left\langle [\{\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})\} \cdot \hat{\mathbf{r}}]^3 \right\rangle = -\frac{4}{5}\epsilon_u r, \qquad (1)$$

where $\hat{\mathbf{r}}$ is the unit vector along the vector $\mathbf{r}$, and r lies in the inertial range, i.e., $\eta \ll r \ll L$. Under time reversal with $\mathbf{u}' = -\mathbf{u}$, the above equation translates to

$$\left\langle [\{\mathbf{u}'(\mathbf{x}+\mathbf{r}) - \mathbf{u}'(\mathbf{x})\} \cdot \hat{\mathbf{r}}]^3 \right\rangle = \frac{4}{5}\epsilon_u r. \qquad (2)$$

Thus, a flow constructed using $-\mathbf{u}$ exhibits negative energy flux, that is, from small scales to large scales; this is unreal. Hence, energy flux provides a powerful diagnostics

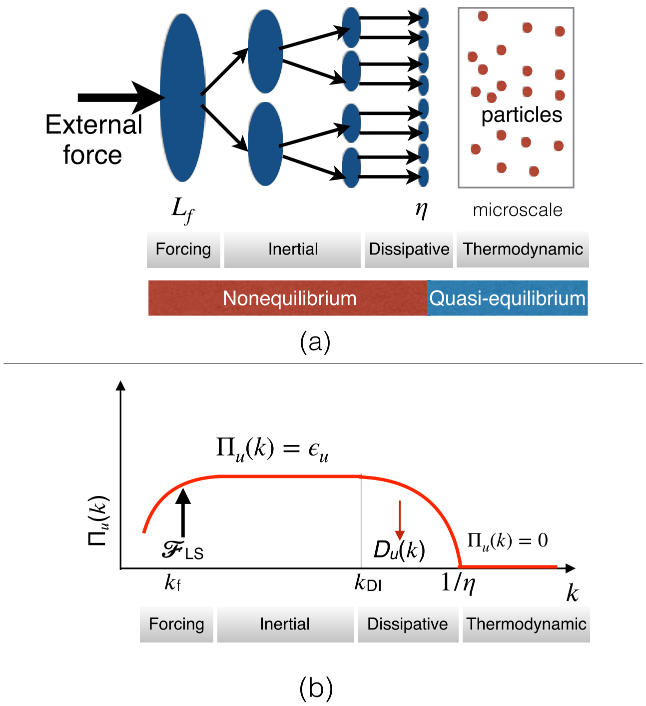

Fig. 1. Schematic diagrams illustrating energy transfers in three-dimensional hydrodynamic turbulence, which is an example of a driven-dissipative nonequilibrium system. (a) The energy supplied at large scales cascades to the inertial range and then to the dissipative range. The dissipation of coherent kinetic energy at the dissipative scales heats up the particles at microscale. The energy transfers are indicated by arrows. The structures at the forcing and inertial range are governed by nonequilibrium processes, while those at microscale are in quasi-equilibrium. (b) Plot of spectral energy flux $\Pi_u(k)$ vs. k. $\Pi_u(k) \sim$ const in the inertial range, and it decreases with k in the dissipative range. $\mathcal{F}_{\rm LS}$ is the energy supply by the external force, and $D_u(k)$ is the dissipation rate. There is no energy flux at the microscales due to the detailed balance.

that can help us differentiate forward and time-reversed flows.

It is customary to describe turbulence in Fourier space too with wavenumber $k \sim 1/l$. In this description, as shown in Figure 1b, energy fed at the large scales (at $k_f = 1/L_f$) cascades to intermediate and small scales. The energy flux (or energy cascade rate) $\Pi_u(k) \sim$ const in the inertial range, and it dampens in the dissipative range $k_{\rm DI}<k<1/\eta$. There is no energy flux in the microscopic range.

The nonlinear term of the Navier-Stokes equation, $\mathbf{u}\cdot\nabla\mathbf{u}$ where $\mathbf{u}$ is the velocity field, induces energy transfers from one scale to another scale. In Fourier space, the basic unit of nonlinear interactions is a wavenumber triad $(\mathbf{k},\mathbf{p},\mathbf{q})$ satisfying $\mathbf{k}=\mathbf{p}+\mathbf{q}$. Dar et al. [27] and Verma [28] showed that the energy transfer from Fourier mode $\mathbf{u}(\mathbf{p})$ to Fourier mode $\mathbf{u}(\mathbf{k})$ under the mediations of Fourier mode $\mathbf{u}(\mathbf{q})$ is given by

$$S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = \Im\left[\{\mathbf{k}\cdot\mathbf{u}(\mathbf{q})\}\{\mathbf{u}(\mathbf{p})\cdot\mathbf{u}^*(\mathbf{k})\}\right]. \quad (3)$$

Using the above function we define the energy flux $\Pi_u(k_0)$, which is the energy emanating from a wavenumber sphere of radius k_0 due to nonlinear interactions, as

$$\Pi_u(k_0) = \sum_{|\mathbf{p}|\le k_0}\sum_{|\mathbf{k}|>k_0} S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q}). \quad (4)$$

A nonzero flux implies that the phases of the Fourier modes are coherently organised. The flux would be zero if the phases of the Fourier modes are random and uncorrelated. Kolmogorov [12–16] showed that in the inertial range, the energy spectrum and flux are given by

$$E_u(k) = K_{\rm Ko}\epsilon_u^{2/3}k^{-5/3}, \quad (5)$$
$$\Pi_u(k) = \epsilon_u, \quad (6)$$

where $k=|\mathbf{k}|$, and $K_{\rm Ko}$ is Kolmogorov's constant whose value is approximately 1.6 [12]. Pao [29] constructed a model to generalise the above to the dissipation rate that predicts a multiplication factor of $\exp(-(k\eta)^{4/3})$ to $E_u(k)$ and $\Pi_u(k)$ of equations (5) and (6). These spectrum and flux have been verified in a large number of experiments and simulations [12,15,16,30].

Now we investigate in some detail the energy transfers and flux in a 3D turbulent flow. Using perturbative field theory to first order, Verma and coworkers [28,31,32] computed averaged $S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})$ as

$$\begin{aligned}\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle = \frac{1}{\rm denr}&[k'\sin(\beta-\gamma)C(\mathbf{p})C(\mathbf{q})\\ &+p\sin(\gamma-\alpha)C(\mathbf{k}')C(\mathbf{q})\\ &+q\sin(\alpha-\beta)C(\mathbf{k}')C(\mathbf{p})\\ &+\sin\beta C(\mathbf{q})(C(\mathbf{k}')-C(\mathbf{p}))], \end{aligned}\quad (7)$$

where α,β, and γ are the internal angles of the triangle formed by $(\mathbf{k},\mathbf{p},\mathbf{q})$, $C(\mathbf{k}')$ is the modal kinetic energy, and

$${\rm denr} = \nu(\mathbf{k})k^2 + \nu(\mathbf{p})p^2 + \nu(\mathbf{q})q^2 \quad (8)$$

with $\nu(\mathbf{k})$ as the renormalized viscosity. Note that $\nu(\mathbf{k})$ plays a fundamental role in turbulence [12,15,33], and in time irreversibility. For isotropic and homogeneous turbulence, and in the inertial range, we substitute

$$C_u(\mathbf{k}) = \frac{E_u(k)}{4\pi k^2} = \frac{K_{\rm Ko}}{4\pi}\epsilon_u^{2/3}k^{-11/3}, \quad (9)$$
$$\nu(\mathbf{k}) = \nu_*\sqrt{K_{\rm Ko}}\epsilon_u^{1/3}k^{-4/3} \quad (10)$$

in equation (7) and obtain numerical values of $\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle$. Domaradzki and Ragallo [34], and Zhou [35] derived equivalent formulas for the energy transfers using different techniques.

Figure 2 exhibits the density plot of $\langle S^{uu}(\mathbf{k}'|\mathbf{p}|\mathbf{q})\rangle$ in logarithmic scale for the inertial range wavenumbers. Here, x and y axes represent p/k and q/k respectively. We observe that in most regions [28,31,32,34,35],

$$\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle > 0 \text{ for } p/k < 1, \quad (11)$$
$$\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle < 0 \text{ for } p/k > 1. \quad (12)$$

Hence, energy flows preferentially from smaller k (large scale) to larger k (small scales), thus breaking the time reversal symmetry as well as the detailed balance of the

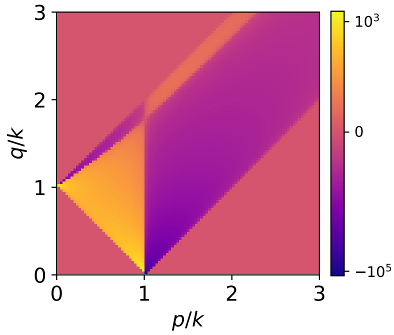

Fig. 2. For 3D hydrodynamic turbulence, density plot of mode-to-mode energy transfer $\langle S^{uu}(\mathbf{k}'|\mathbf{p}|\mathbf{q})\rangle$ computed using perturbative field theory to first order. Figure indicates significant energy transfer from small p to k.

energy transfer. Substitution of $\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle$ of equation (7) in equation (4) yields [12,15,28,32]

$$\Pi_u(k) = \epsilon_u \tag{13}$$

for the inertial range wavenumbers.

The energy flux, which is a sum of S^{uu}'s (Eq. (6)), is positive in the inertial range [12,15,32]. These energy transfers create and sustain a hierarchy of structures.

Note that equation (7) yields the same energy transfers for both $\mathbf{u}$ and $-\mathbf{u}$ flow profiles, unlike equations (2) and (3) that change sign under this operation. This is due to the *random approximation* and fully-developed nature of turbulence employed for the field-theoretic analysis during the derivation of equation (7). This averaging may be similar to the molecular chaos hypothesis of Boltzmann's H-theorem.

Under the time reversal transformation $\mathbf{u} \to -\mathbf{u}$, the mode-to-mode energy transfer $S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})$ of equation (3) changes sign, similar to that in time-reversed structure function (see Eq. (2)). Thus, the energy transfer and flux differentiate the real flow with the time-reversed velocity field. Note however that the negative energy flux for $-\mathbf{u}$ is a transient behaviour. When we time advance this flow further, after a short while, the flow readjusts itself to yield a forward energy transfer, which is the natural direction of energy transfer (see Eqs. (7) and (13)).

For a steady incompressible turbulent flow, the energy contents of the structures at the large and intermediate scales remain unchanged on an average. Hence, the measure of disorder (entropy) is not expected to increase for these structures. However, the energy supplied at the large scales is finally transferred to the molecules at microscales. Hence the molecules at the microscopic scales get heated up, thus increasing the entropy at this level (see Fig. 1). The net increase in the entropy at the microscales, ΔS, would be

$$\Delta S = \frac{1}{T}\int \epsilon_u dt, \tag{14}$$

where T is the average temperature of the fluid. Thus, on the whole, physics of turbulence is consistent with the second law of thermodynamics.

In summary, we argue that the unidirectional energy transfer in turbulence provides another perspective on the time reversal asymmetry in macroscopic systems. Note that the structures in forcing and inertial range, as well as those of dissipation range, are in nonequilibrium. However the structures of dissipation range and all of microscopic range are in quasi-equilibrium due to slow heating. See Figure 1 for an illustration, and a recent book by Chibbaro et al. [36] for some more discussion on similar topic.

There are exceptions to the above scenario of energy transfers. The energy tends to flow from intermediate scales to large scales in two-dimensional turbulence, quasi two-dimensional turbulence, helical turbulence, and in rapidly rotating turbulence. In addition, the energy flux vanishes for Euler turbulence (Navier-Stokes equation with $\nu = 0$). We will describe these systems in Section 5.

In the next section, we argue that the above picture is applicable to many other nonequilibrium systems exhibiting multiscale physics.

3 Generalization to other nonequilibrium systems

The aforementioned energy transfers from large scales to small scales are observed in many driven-dissipative nonequilibriun systems – earthquakes [37], magnetohydrodynamic turbulence [28,38], scalar turbulence and thermal convection [39–41], finance [42], and astrophysical and geophysical flows [43–45]. In thermal convection, large-scale thermal plumes make the flow turbulent. Galactic and stellar turbulence have behaviour similar to hydrodynamic turbulence with energy feed at large scale by supernovas and star core (where nuclear reaction takes place) respectively [44]. Collisions of tectonic plates feed energy to earthquakes; this energy is transmitted to smaller scales. The money supply at large scales drives a free market economy.

The energy transfers in the above systems differ in detail, but energy flows from large scales to small scales in all of them. This energy is dissipated at small scales. Hence, this asymmetric energy transfer provides an interesting viewpoint for the arrow of time in such nonequilibrium systems. However, details of energy transfers need to be worked out for such systems; this exercise will be taken up in future.

Another important question is how to introduce an effective dissipation in a nonequilibrium system. We observe that the physics of turbulence provides important clues for the same, as described in the next section.

4 How to incorporate dissipation in a physical system?

Fundamental forces of nature are conservative or nondissipative. Hence, an inclusion of dissipation appears impossible from the perspectives of fundamental physics. Yet, multiscale description of turbulence provides an interesting possibility for an inclusion of dissipation in a physical system.

An important question: what is the origin of viscous dissipation in a fluid flow when it is composed of many interacting molecules and atoms, as visualised in statistical mechanics? Naively, we do not expect any dissipation in a flow. The answer however becomes apparent from multiscale perspective with a separation of coherent and incoherent structures. As shown in Figure 1, in a turbulent flow, the relative velocity between the fluid structures causes successive cascade of coherent energy to smaller and smaller scales. Finally, the coherent structures are fully dissipated at Kolmogorov's microscale (η). Hence, in hydrodynamic turbulence, viscous dissipation provides an interesting model for transferring energy from coherent structures to incoherent ones. We emphasize that in a decaying turbulence (with no external force), the total kinetic energy of the flow, $\rho u^2/2$ where ρ is the fluid density, decays. But the total energy, which is a sum of kinetic energy of the fluid and particles, is conserved. Thus, the viscosity converts coherent kinetic energy to thermal energy at microscales. Similar processes occur in plasma turbulence where the energy supplied from the large-scale velocity and magnetic fields are dissipated at microscales by kinetic processes involving charge particles [46,47].

We expect a similar separation of scales in many driven-dissipative nonequilibrium systems – astrophysical turbulence (in stars, galaxies, galaxy clusters, etc.), free market economy with cascade of money, earthquakes, etc. The energy (or similar quantity, e.g. money in a financial system) flows from large scales to small scales, where it is dissipated. The physical processes at small scales that converts coherent energy to incoherent ones could be treated as dissipation. Even the frictional loss during the motion of a block on a surface can be treated as conversion of coherent kinetic energy of the block to the incoherent heat energy of molecules at the interface. Introduction of dissipation in the universe and many-body (macroscopic) quantum systems appears to be a possibility from this perspective.

The universe is very complex due to intricate spacetime structures, complicated and enormous energy sources and sinks, unknown initial condition (during the early universe), etc. Gravitational interactions are conservative. Yet, from hydrodynamic perspectives, it is possible to treat the universe as a driven-dissipative nonequilibrium system with energy flowing from large scales (driven by sources like supernova) to intermediate scales and then to small scales. For example, large-scale kinetic energy of a galaxy is transferred to the motion of dust and/or charged particles. Such mechanisms are employed for the generation of magnetic field in the universe [44]. Note however that dynamical interactions among the entities could lead to formation or destruction of structures. For example, depending on the forces, (local) space dimensionality, and initial configurations, the flow could become less structured or more structured. Note that large-scale structures are formed in two-dimensional or quasi-two-dimensional turbulence [12,15,21,48,49]. Hence, the aforementioned dynamical and multiscale perspective with energy transfers could possibly provide a cosmological arrow of time for the universe (even in the scenario of collapsing universe).

Dissipation or decoherence in quantum systems is quite intriguing, and it remains primarily an unsolved problem. Some of the proposed mechanisms for quantum decoherence are collapse of wave function during a measurement, interaction with enviornment [10], chaos [11], etc. In this short article, we do not delve into these topics, but describe recent observations on dissipation in quantum turbulence. Recent experiments and numerical simulations on superfluid turbulence reveal that for a wavenumber band, $E_u(k) \sim k^{-5/3}$ with a constant energy cascade [50], similar to hydrodynamic turbulence. Bradley et al. [51] argued that the energy dissipation in Helium-3 superfluid turbulence, which is quantum many-body system, occurs via phonon coupling. Thus microscopic processes like phonon interactions in Helium-3 could provide dissipation of large-scale quantum correlations (also see [52]). It would be interesting to attempt similar ideas for other many-body quantum systems, such as Bose-Einstein condensate, quantum cavity with many atoms, etc.

Thus, a conversion of (cascaded) coherent energy to incoherent one at small scales provides an interesting way to incorporate dissipation in a driven-dissipative nonequilibrium system, as well as in many-body classical and quantum systems. This idea however needs further explorations using experiments and numerical simulations.

5 On systems exhibiting inverse energy cascade and no energy cascade

Interestingly there are exceptions to aforementioned picture of energy cascade from large scales to small scales. Two-dimensional (2D) and quasi two-dimensional turbulence, rotating turbulence, and helical turbulence exhibit inverse cascade of energy [15,21,48,49,53]. That is, in such systems, the energy flows from intermediate scales to large scales. For such systems too, the energy transfer provides an interesting perspective on the arrow of time. For the same we dig into 2D turbulence in some detail.

The inverse cascade in 2D turbulence is due to the nature of nonlinear interactions. Here, for the inertial range with inverse energy cascade, field-theoretic computations of $\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle$ (corresponding to Eq. (7)) yield [28,31,32]

$$\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle < 0 \text{ for } p/k < 1, \tag{15}$$
$$\langle S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q})\rangle > 0 \text{ for } p/k > 1, \tag{16}$$

which is opposite to those for 3D turbulence. Hence, in 2D turbulence, the energy transfer is from intermediate scales to large scales. For such flows, time-reversed configuration ($\mathbf{u} \to -\mathbf{u}$) would yield forward energy flux, which is contrary to the real 2D flows. Hence, the energy transfer computations helps us in determining the arrow of time for 2D turbulence as well. It is just that we need to incorporate the space dimensionality factor in energy transfers. Similar arguments hold for rotating and helical turbulence [53].

An important point to note that in the above flows, the energy is dissipated at all scales, including the large scales.

For example, in 2D turbulence, dissipation at large scales is modelled using Ekman friction. Consequently, from the perspectives of dissipation, the entropy of the system increases with time. Note however that the increased strength of structures in such flows may lead us to a contrary conclusion that the entropy is decreasing. These issues however need closer scrutiny.

Now we take up Euler flow that has zero energy flux. The Euler flow is a fluid flow with $\nu = 0$. A widely studied model of Euler turbulence is a truncated one containing a finite number of interacting Fourier modes. It has been conjectured that the truncated Euler equation obeys Liouville's theorem and ergodic hypothesis. Therefore, all the Fourier mode have equal energy (statistically) [15], and hence a substitution of $C(\mathbf{k}) = C(\mathbf{p}) = C(\mathbf{q})$ in equation (7) yields $S^{uu}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = 0$ [28,32]. Thus, the truncated Euler equation is in equilibrium with zero energy flux. An equilibrium configuration tends to indicate statistically time-invariant flow with maximum entropy. Yet, the flow has no dissipation because $\nu = 0$. Such dichotomies are possibly due to the multiscale nature of the flow, as well as generalised view of the equilibrium, and they need a detailed examination.

In addition there are systems that are dissipative with no net energy flux, for example laminar flows. In such systems, the energy flows from large scale structures to the dissipative scales. Such energy transfers increase the entropy of the system. Here too, the direction of energy transfer is related to the arrow of time.

We conclude in the next section.

6 Conclusions and discussions

In summary, unidirectional and multiscale energy transfers in turbulence and in driven-dissipative nonequilibrium systems provide another perspective on the arrow of time in such systems. In this framework, the dissipation at small scales and energy supply at large scales play a critical role. An important outcome of such formalism is a recipe to introduce dissipation in multiscale systems. We propose that the transformation from coherent energy to incoherent energy could be treated as dissipation in physical systems.

Interestingly, many biological systems too exhibit multiscale processes with energy supply at large scales, sustenance at intermediate scale, and dissipation at small scales. For example, a living being receives food at large scales, which is used by organs (all the way to cells in a hierarchal manner) to generate energy and nutrition at intermediate scales. The waste products (e.g. CO_2) generated at small scales are excreted out. These time-asymmetric processes may be playing a key role in determining biological arrow of time.

Thus, energy transfers in turbulence and other nonequilibrium flows provide interesting perspectives on the arrow of time. As described in Section 2, we can determine whether a given flow profile is real or time-reversed using energy transfer computations. Three-dimensional turbulence exhibits forward energy cascade, while time-reversed flow profile shows negative energy cascade for a short while. In comparison, entropy of a single configuration cannot provide this distinction; we need entropy of two snapshots to determine which way the time is moving.

We hope that the aforementioned multiscale framework may be useful for resolving some of the longstanding issues on the arrow of time in physical and biological systems, as well as in cosmology.

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Author contribution statement

I, as a single author, was involved in the preparation of the manuscript. I have read and approve the final manuscript.

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