Scalings of field correlations and heat transport in turbulent convection

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Using direct numerical simulations of Rayleigh-Bénard convection under free-slip boundary condition, we show that the normalized correlation function between the vertical velocity field and the temperature field, as well as the normalized viscous dissipation rate, scales as $\mathrm{Ra}^{-0.22}$ for moderately large Rayleigh number Ra. This scaling accounts for the Nusselt number Nu exponent of approximately 0.3, as observed in experiments. Numerical simulations also reveal that the aforementioned normalized correlation functions are constants for the convection simulation under periodic boundary conditions.

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Thermal convection, which is ubiquitous in engineering and natural flows, exhibits various interesting phenomena such as instabilities, chaos, spatiotemporal patterns, and turbulence [1,2]. Rayleigh-Bénard convection (RBC) is an idealized model of thermal convection in which a fluid is placed between two horizontal conducting plates, with the lower plate being hotter than the top one. The dynamics of the flow is governed by two nondimensional parameters: the Rayleigh number Ra, a measure of the strength of the buoyancy force, and the Prandtl number Pr, a ratio of the kinematic viscosity and the thermal diffusivity. One of the most important theoretical and technological problems that remains unsolved in this field is the nature of the correlation between temperature and velocity field and consequently its effect on the behavior of heat transport in RBC, especially for very large Rayleigh number; this is the subject matter of this paper.

Experiments and numerical simulations reveal certain universal properties for the heat flux [1–6]. Various experiments show that the Nusselt number, which is the ratio of the total (convective plus conductive) heat flux to the conductive heat flux, scales as Ra^{β} with β approximately 0.3 for moderately large Rayleigh numbers [7–14]. However, for very high Rayleigh numbers (called the ultimate regime), Kraichnan [3] predicted that $\beta = 1/2$; several experiments found no evidence for the ultimate regime [7–12], while some others claimed its existence [13,14]. In this paper we show that the velocity-temperature correlation and the viscous dissipation rate vary with the Rayleigh number so as to yield a Nusselt number exponent of 0.3 for intermediate Rayleigh numbers.

One of the earliest attempts to understand Nusselt number scaling was by Kraichnan [3], who used the mixing-length theory to derive that $\mathrm{Nu} \propto \mathrm{Ra}^{1/3}$ for large Pr, $\mathrm{Nu} \propto (\mathrm{PrRa})^{1/3}$ for small Pr, and $\mathrm{Nu} \sim 1$ for very small Pr. For very large Rayleigh number or the ultimate regime, Kraichnan [3] showed that $\mathrm{Nu} \propto (\mathrm{Ra}/\ln \mathrm{Ra})^{1/2}$. Malkus [4] argued that the Nu exponent of 1/3 is due to the property that the heat flux is independent of the cell height. By separating the dissipation rates in the bulk and the boundary layers, Grossmann and Lohse [5] showed that for the bulk-dominated convective flows, $\mathrm{Nu} \sim (\mathrm{PrRa})^{1/2}$ when $\lambda_u < \lambda_\theta$, but $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$ when $\lambda_u > \lambda_\theta$. Here λ_u and λ_θ are the widths of the viscous and thermal boundary layers, respectively. The parameter space of validity for the above scaling has been detailed by Grossmann and Lohse [5]. Shraiman and Siggia [6], Castaing *et al.* [7], and Cioni *et al.* [8] computed the Nu exponent and deduced it to be 2/7 due to the boundary layers. Using scaling arguments, Verzicco and Camussi [15] claimed that $\mathrm{Nu} \sim (\mathrm{Ra})^{1/4}$ for small Pr ($\mathrm{Pr} < 1$).

Many experiments have been performed to test the above scaling, yet they have not been able to resolve the scaling exponents completely. Laboratory experiments on typical fluids, water, helium gas, and mercury yield Nusselt number exponents from 0.26 to 0.31 for Ra up to 10^{17} [7–12]. Cioni *et al.* [8] and Glazier *et al.* [10] used mercury, while Castaing *et al.* [7] and Niemela *et al.* [9] used helium gas as their experimental fluid. Cioni *et al.* [8] also performed experiments on water for comparison. Funfschilling *et al.* [12] employed a pressurized mixture of gas in their experiment. Figure 1 illustrates the plots of reduced Nusselt number $\mathrm{Nu}/(\mathrm{PrRa})^{0.27}$ vs PrRa computed in earlier RBC experiments and numerical simulations. Chavanne *et al.* [13] and Roche *et al.* [14] reported the existence of the ultimate regime in their experiment on helium. Roche *et al.* used nonsmooth surfaces to cancel the thickness variation of the viscous sublayer. Funfschilling *et al.* [12] found the simultaneous existence of multiple Nusselt number exponents (0.17, 0.25, and 0.36) for large Ra, possibly due to multiple coexisting attractors.

The Nusselt number for RBC is also investigated using direct numerical simulation. Verzicco and Camussi [15] and Kerr and Herring [16] showed that $\mathrm{Nu} \sim \mathrm{Ra}^{1/4}$ for small Pr, while Silano *et al.* [17], Stevens *et al.* [18], and Verzicco and Sreenivasan [19] reported the exponent to be approximately 1/3 for very high Rayleigh number. Kerr [20] and Kerr and Herring [16] found the exponent to be approximately $0.28 \approx 2/7$ for larger Pr. Figure 1 illustrates some of these results, as well as our numerical results for $\mathrm{Pr} = 0.2$ and 6.8 (to be described later). Kerr [21] also studied the energy budget in RBC by computing the mean-square velocity and dissipation rates as a function of Rayleigh number for various aspect ratios and Prandtl numbers.

For bulk-dominated convective flows, a simple argument would predict that $\mathrm{Nu} \propto \langle u_z \theta \rangle \sim \mathrm{Ra}^{1/2}$ (with θ the temperature fluctuation). This scaling argument assumes that $\langle u_z^2 \rangle^{1/2} \sim \mathrm{Ra}^{1/2}$ and $\langle \theta^2 \rangle^{1/2} \sim$ const and ignores the correlation between the vertical velocity field and the temperature field. Similarly,

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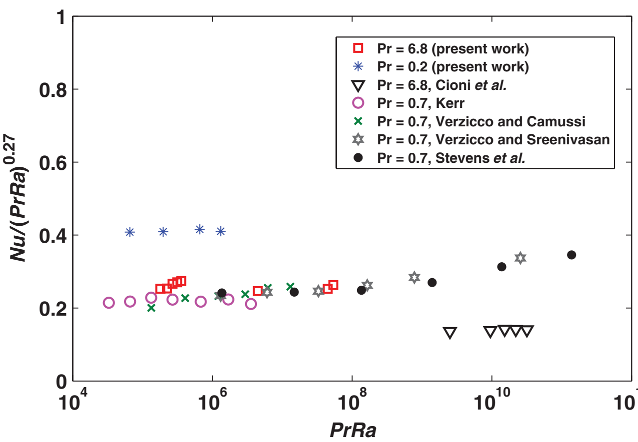

FIG. 1. (Color online) Reduced Nusselt number $\mathrm{Nu}/(\mathrm{PrRa})^{0.27}$ versus PrRa, the product of the Prandtl number Pr and Rayleigh number Ra. Experimental data are from Cioni *et al.* [8] for water (black $\triangledown$). Numerical data are from Verzicco and Camussi [15] for $\mathrm{Pr} = 0.7$ (green $\times$), Kerr [20] for $\mathrm{Pr} = 0.7$ (magenta $\circ$), Stevens *et al.* [18] for $\mathrm{Pr} = 0.7$ (black $\bullet$), Verzicco and Sreenivasan [19] for $\mathrm{Pr} = 0.7$ (gray ✡), and our simulation data for $\mathrm{Pr} = 6.8$ (red $\Box$) and $\mathrm{Pr} = 0.2$ (blue $*$).

one of the exact relations $\mathrm{Nu} - 1 \sim \epsilon^u/\mathrm{PrRa}$ [6] yields $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ if we replace $\epsilon^u \sim U_L^3/L$ while ignoring correlations (U_L is the large scale velocity, which is the free-fall velocity). A major difficulty in this field is how to reconcile the $\mathrm{Nu} \sim \mathrm{Ra}^{0.3}$ scaling observed in the experiments to the $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ predicted for the ultimate regime. In this paper, we will explicitly compute the velocity-temperature correlation function and show that it scales with Ra in such a way that $\mathrm{Nu} \sim \mathrm{Ra}^{0.3}$ at moderate Rayleigh numbers.

Properties of convective flow depend quite critically on the boundary layers [22]. However, the Nu scaling appears to be somewhat insensitive to the presence of boundary layers [23] and a change of the boundary conditions [19]. Motivated by the above observations, we attempt to compute the scaling of the Nusselt and Péclet numbers in a turbulent regime in terms of bulk quantities, in particular the large-scale velocity U_L and the large-scale temperature θ_L $(= \sqrt{\langle\theta^2\rangle})$. The study of the combined effects of the bulk and boundary layers requires more refined simulations and theoretical arguments and will not be discussed herein.

The RBC equations under the Boussinesq approximation for a fluid placed between two plates, separated by distance d and with a temperature difference of Δ, are

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\nabla\sigma}{\rho_0} + \alpha g\theta\hat{\mathbf{z}} + \nu\nabla^2\mathbf{u}, \tag{1}$$

$$\partial_t \theta + (\mathbf{u}\cdot\nabla)\theta = \frac{\Delta}{d}u_z + \kappa\nabla^2\theta, \tag{2}$$

where θ and σ are, respectively, the temperature and pressure fluctuations from the steady conduction state ($T = T_c + \theta$, with T_c the conduction temperature profile); $\hat{\mathbf{z}}$ is the buoyancy direction; ρ_0 is the mean density of fluid; g is the acceleration due to gravity; and α, ν, and κ are the thermal heat expansion coefficient, the kinematic viscosity, and the thermal diffusivity of fluid, respectively. The two most important nondimensional parameters of RBC are the Rayleigh number $\mathrm{Ra} = \alpha g \Delta d^3/\nu\kappa$ and the Prandtl number $\mathrm{Pr} = \nu/\kappa$. The Nusselt number can be expressed as

$$\mathrm{Nu} = \frac{\kappa\Delta/d + \langle u_z T\rangle}{\kappa\Delta/d} = 1 + \left\langle \frac{u_z d}{\kappa}\frac{\theta}{\Delta}\right\rangle = 1 + \langle u_z'\theta'\rangle, \tag{3}$$

where the nondimensionalized vertical velocity and temperature fields are $u_z' = u_z d/\kappa$ and $\theta' = \theta/\Delta$, respectively.

Equation (2) has three competing terms: $(\mathbf{u}\cdot\nabla)\theta$, $\frac{\Delta}{d}u_z$, and $\kappa\nabla^2\theta$. When the diffusion term $\kappa\nabla^2\theta$ of Eq. (2) is much smaller than the other two, we obtain

$$\frac{U_L\theta_L}{d} \approx \frac{(u_z)_L\Delta}{d} \quad \Rightarrow \theta_L \approx \Delta. \tag{4}$$

In the momentum equation [Eq. (1)]

$$\frac{U_L^2}{d} \approx \alpha g\theta_L \Rightarrow U_L \approx \sqrt{\alpha g\Delta d}, \tag{5}$$

implying that the Péclet number $\mathrm{Pe} = U_L d/\kappa$ scales as $\mathrm{Pe} \approx \sqrt{\mathrm{PrRa}}$. These scaling relations are applicable when $(u_z)_L\Delta/d \gg \kappa\nabla^2\theta$ or $\mathrm{PrRa} \gg 1$. These results are consistent with the predictions of Kraichnan [3], Grossmann and Lohse [5], and Silano *et al.* [17].

Note, however, that for $\mathrm{PrRa} \ll 1$, the diffusion term dominates the nonlinear term and is balanced by $u_z\Delta/d$. Therefore, $\theta_L \approx U_L d\Delta/\kappa$ and $\mathrm{Pe} \approx \mathrm{PrRa}$. Under this condition, $U_L \propto \theta_L$; hence $\mathrm{Nu} = 1 + c(\mathrm{PrRa})^2$, with c a constant. Convective turbulence with the $\mathrm{PrRa} \ll 1$ limit is rarely observed in terrestrial experiments or astrophysical observations. Therefore, in the present paper we limit ourselves to the $\mathrm{PrRa} \gg 1$ regime. In Fig. 2 we plot the reduced Péclet number $\mathrm{Pe}/(\mathrm{PrRa})^{0.5}$ computed in various RBC experiments and numerical simulations; these results are in general agreement with $\mathrm{Pe} \sim (\mathrm{PrRa})^{1/2}$ scaling [24]. The data of Figs. 1 and 2 appear to be compactified reasonably well with PrRa, hence we use PrRa as a scaling variable.

As discussed earlier, a naive replacement of $\langle u_z'\theta'\rangle$ of Eq. (3) with $\langle u_z'^2\rangle^{1/2}\langle\theta'^2\rangle^{1/2}$ yields $\mathrm{Nu} \sim \sqrt{\mathrm{PrRa}}$, which is

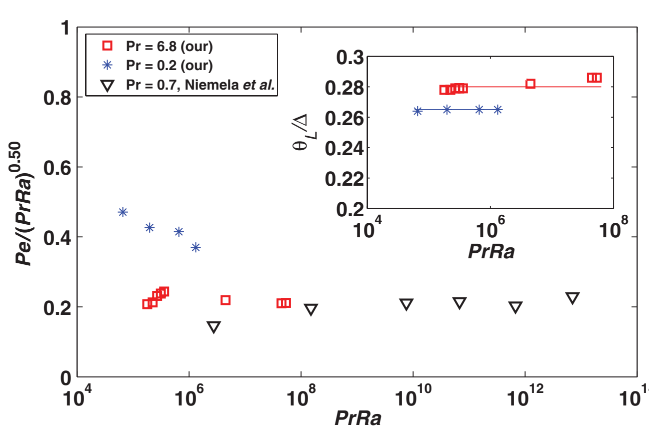

FIG. 2. (Color online) Reduced Péclet number $\mathrm{Pe}/(\mathrm{PrRa})^{0.5}$ versus PrRa. Black $\triangledown$ represent the experimental data for helium ($\mathrm{Pr} \sim 0.7$) by Niemela *et al.* [24]. Red $\Box$ and blue $*$ represent the present simulation data for $\mathrm{Pr} = 6.8$ and 0.2, respectively. The inset shows the constancy of the large-scale temperature field θ_L/Δ with PrRa for $\mathrm{Pr} = 6.8$ (red $\Box$) and 0.2 (blue $*$).

not observed in the experiments and simulations for moderately large Rayleigh numbers (see Fig. 1). To account for the velocity-temperature correlation, we rewrite the Nusselt number as

$$\mathrm{Nu} - 1 = \langle u_z'\theta'\rangle = C^{u\theta}(\mathrm{PrRa})\langle u_z'^2\rangle_V^{1/2}\langle\theta'^2\rangle_V^{1/2}, \qquad (6)$$

where

$$C^{u\theta}(\mathrm{PrRa}) = \left\langle \frac{\langle u_z'\theta'\rangle_V}{\langle u_z'^2\rangle_V^{1/2}\langle\theta'^2\rangle_V^{1/2}} \right\rangle_t$$

is the normalized correlation function between the vertical velocity and temperature, with $\langle\,\rangle_V$ and $\langle\,\rangle_t$ representing the volume and temporal averages, respectively. Note that $C^{u\theta}(\mathrm{PrRa}) \leqslant 1$ as a consequence of the Cauchy-Schwarz inequality. We perform numerical simulations to compute the above correlation, along with the Péclet number, Nusselt number, and θ_L, for various Pr and Ra values.

We numerically solve Eqs. (1) and (2) using the pseudospectral method on grid sizes ranging from 128^3 to 512^3. We apply free-slip boundary conditions for the velocity field and isothermal boundary conditions for the temperature field. We use the Runge-Kutta fourth-order scheme for time advancement and compute the relevant quantities in the steady state. For further details of the simulation, refer to Refs. [25,26].

In Figs. 1 and 2 we plot the computed reduced Nusselt and Péclet numbers vs PrRa for Pr = 6.8 (red squares) and 0.2 (blue asterisks). The best fits for our data are $\mathrm{Nu} = (0.27 \pm 0.04)(\mathrm{PrRa})^{0.27\pm0.01}$ and $\mathrm{Pe} = (0.26 \pm 0.04)(\mathrm{PrRa})^{0.49\pm0.01}$ for $\mathrm{Pr} = 6.8$, and $\mathrm{Nu} = (0.39 \pm 0.02)(\mathrm{PrRa})^{0.27\pm0.01}$ and $\mathrm{Pe} = (1.04 \pm 0.20)(\mathrm{PrRa})^{0.43\pm0.02}$ for $\mathrm{Pr} = 0.02$. The inset of Fig. 2 demonstrates the constancy of θ_L/Δ with PrRa. These results are consistent with the earlier experimental and numerical observations, as shown in the figures.

We compute the normalized correlation function $C^{u\theta}(\mathrm{PrRa})$ for Pr = 6.8 and various Ra values and plot them in Fig. 3. We observe that

$$C^{u\theta}(\mathrm{PrRa}) \approx (5.6 \pm 1.30)(\mathrm{PrRa})^{-0.22\pm0.017} \qquad (7)$$

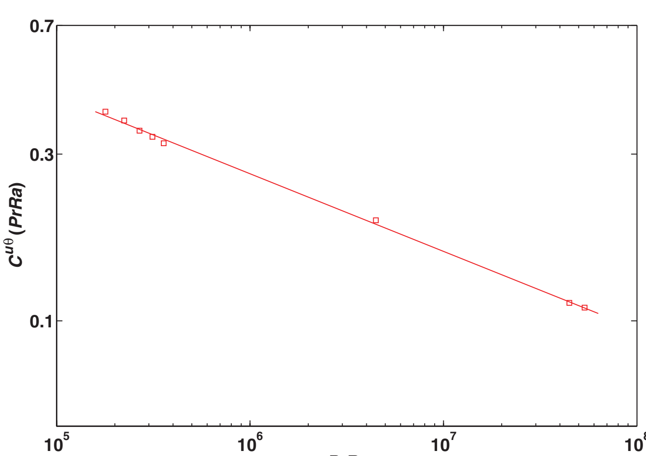

FIG. 3. (Color online) Normalized correlation function $C^{u\theta}(\mathrm{PrRa}) = \langle \langle u_z'\theta'\rangle_V / \langle u_z'^2\rangle_V^{1/2}\langle\theta'^2\rangle_V^{1/2}\rangle_t$ versus PrRa. The function $(5.6 \pm 1.30)(\mathrm{PrRa})^{-0.22\pm0.017}$ fits well with the simulation data.

for $\mathrm{Ra} \lessapprox 10^8$. The above correlation is possibly due to the interactions between the bulk flow and the boundary layers or the interactions of the large-scale flows. However, its power-law behavior is interesting, which immediately yields $\mathrm{Nu} \sim \mathrm{Ra}^{0.27}$, which is consistent with the Nu scaling observed in experiments and numerical simulations. Also, a combination of Eqs. (6) and (7) provides

$$\mathrm{Nu} - 1 = 0.046\{C^{u\theta}(\mathrm{PrRa})\}(\mathrm{PrRa})^{1/2}. \qquad (8)$$

We can also deduce another important field correlation using one of the exact relations for the RBC flow [6]:

$$\mathrm{Nu} - 1 = \frac{\mathrm{Pr}^2 d^4 \epsilon^u}{\nu^3 \mathrm{Ra}} = \frac{(\mathrm{Pe})^3}{\mathrm{PrRa}} C^{\epsilon^u}(\mathrm{PrRa}), \qquad (9)$$

where the dissipation rate of the kinetic energy $\epsilon^u = (U_L^3/d)C^{\epsilon^u}(\mathrm{PrRa})$, with the function $C^{\epsilon^u}(\mathrm{PrRa})$, called the normalized viscous dissipate rate, playing a role similar to that of the function $C^{u\theta}(\mathrm{PrRa})$. Since $\mathrm{Pe} = a(\mathrm{PrRa})^{1/2}$, $C^{\epsilon^u}(\mathrm{PrRa})$ must scale as $s(\mathrm{PrRa})^{-0.22}$, similar to $C^{u\theta}(\mathrm{PrRa})$ (here a and s are constants). Thus

$$\mathrm{Nu} - 1 = a^3(\mathrm{PrRa})^{1/2} s(\mathrm{PrRa})^{-0.22} = a^3 s(\mathrm{PrRa})^{0.27}, \qquad (10)$$

which immediately yields $s \approx 15$.

The normalized correlation function $C^{u\theta}(\mathrm{PrRa})$ and the normalized viscous dissipation rate $C^{\epsilon^u}(\mathrm{PrRa})$ decrease with increasing Ra since the fields tend to become more and more turbulent. A question arises as to whether these correlations would continue to decrease and vanish as $\mathrm{Ra} \to \infty$ or would they saturate to some asymptotic values. For very large Rayleigh numbers at which global dissipation rates are mainly bulk dominated, we expect the turbulence to be fully developed and, according to Grossmann and Lohse's and Kraichnan's phenomenological theories for fully developed convective turbulence, $\epsilon^u \sim U_L^3/L$, which may lead to $C^{\epsilon^u}(\mathrm{PrRa}) \sim$ const. However, this behavior of field correlation is based

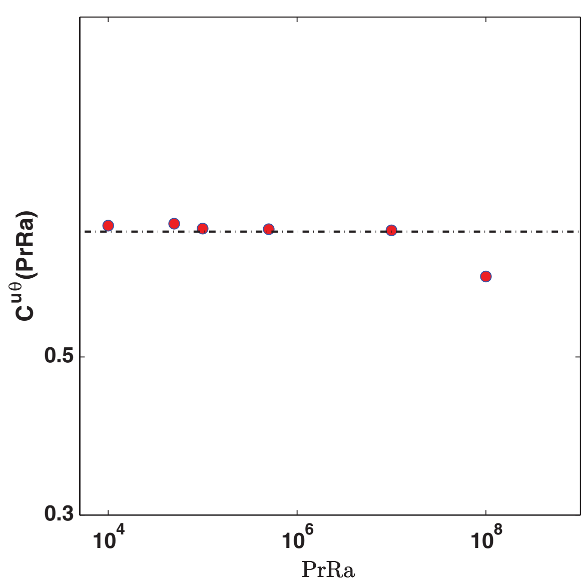

FIG. 4. (Color online) Circles about the dot-dashed line depict the constancy of the normalized correlation function $C^{u\theta}(\mathrm{PrRa})$ for the convection in the periodic box (Pr = 1).

on phenomenological arguments, which needs further support from future numerical and experimental data.

It has been shown that turbulent convection of high Rayleigh numbers or the ultimate regime is somewhat related to the convection with periodic boundary conditions at significantly lower Rayleigh numbers [27]. Following this, we carried out RBC simulations for the periodic box geometry of size $(2\pi)^3$ using a pseudospectral code [25,26]. We computed the normalized correlation function $C^{u\theta}(\mathrm{PrRa})$ for $\mathrm{Pr} = 1$ and high Rayleigh numbers $\mathrm{Ra} = 10^4$–10^8, for which the flow is fully turbulent. We observe that for all the runs, $C^{u\theta}(\mathrm{PrRa}) \approx 0.75$, as shown in Fig. 4, and $C^{\epsilon^u}(\mathrm{PrRa}) \approx 0.48$. In addition, $\mathrm{Pe} \approx 5.7\,\mathrm{Ra}^{0.50\pm,0.02}$, $\mathrm{Nu} \approx 23.7\,\mathrm{Ra}^{0.46\pm0.04}$, and $\theta/\Delta \approx 4.21 \pm 0.21$. Thus the constancy of the field correlations for convection with periodic boundary conditions is consistent with the $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ observed for this case. Furthermore, it strengthens the similarities between the periodic boundary conditions and the ultimate regime. It is also important to note that the flow for the periodic boundary conditions is highly anisotropic with $\langle 2|u_z|^2/(|u_x|^2 + |u_y|^2)\rangle \approx 3.26 \pm 0.43$.

In summary, we relate the Nusselt number scaling to the normalized velocity-temperature correlation function as well as the normalized viscous dissipation rate. We show that these functions scale with the Rayleigh number as $\mathrm{Ra}^{-0.22}$ (approximately) for intermediate Ra, which leads to $\mathrm{Nu} \sim \mathrm{Ra}^{0.3}$, which is observed in experiments. For very large Rayleigh numbers, the flow is expected to be fully bulk dominated, which could affect the field correlations significantly so as to yield $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$, as predicted by Kraichnan [3]. Extensive and difficult numerical simulations and experiments are needed to test this hypothesis. Convection simulations for a periodic box geometry also exhibit constant values for the normalized correlation functions, as well as $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$, which may be related to the field correlations for high-Rayleigh-number turbulent convection. Thus, the field correlations provide valuable insight into the scaling of the Nusselt number.

Numerical simulation of the ultimate regime is beyond the capabilities of present-day computers, but the gap is expected to be bridged soon, which would settle this long-standing problem. In addition, attempts to compute the asymptotic values of the correlations functions using methods similar to the theoretical computations of Doering and co-workers [28] would be valuable.

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[1] E. D. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[2] G. Ahlers, S. Grossmann, and D. Lohse, Rev. Mod. Phys. **81**, 503 (2009); D. Lohse and K. Q. Xia, Annu. Rev. Fluid Mech. **42**, 335 (2010).
[3] R. Kraichnan, Phys. Fluids **5**, 1374 (1962).
[4] M. V. R. Malkus, Proc. R. Soc. London Ser. A **225**, 185 (1954); **225**, 196 (1954).
[5] S. Grossmann and D. Lohse, J. Fluid Mech. **407**, 27 (2000).
[6] B. I. Shraiman and E. D. Siggia, Phys. Rev. A **42**, 3650 (1990).
[7] B. Castaing *et al.*, J. Fluid Mech. **204**, 1 (1989).
[8] S. Cioni, S. Ciliberto, and J. Sommeria, J. Fluid Mech. **335**, 111 (1997).
[9] J. J. Niemela, L. Skerbek, K. R. Sreenivasan, and R. J. Donnelly, Nature (London) **404**, 837 (2000).
[10] J. Glazier, T. Segawa, A. Naert, and M. Sano, Nature (London) **398**, 307 (1999).
[11] P. Urban, V. Musilová, and L. Skrbek, Phys. Rev. Lett. **107**, 014302 (2011).
[12] D. Funfschilling, E. Bodenschatz, and G. Ahlers, Phys. Rev. Lett. **103**, 014503 (2009).
[13] X. Chavanne *et al.*, Phys. Fluids **13**, 1300 (2001).
[14] P.-E. Roche, B. Castaing, B. Chabaud, and B. Hébral, Phys. Rev. E **63**, 045303(R) (2001).
[15] R. Verzicco and R. Camussi, J. Fluid Mech. **383**, 55 (1999).
[16] R. M. Kerr and J. R. Herring, J. Fluid Mech. **419**, 325 (2008).
[17] G. Silano, K. R. Sreenivasan, and R. Verzicco, J. Fluid Mech. **662**, 409 (2010).
[18] R. J. A. M. Stevens, R. Verzicco, and D. Lohse, J. Fluid Mech. **643**, 495 (2010); R. J. A. M. Stevens, D. Lohse, and R. Verzicco, *ibid.* **688**, 31 (2011).
[19] R. Verzicco and K. Sreenivasan, J. Fluid Mech. **595**, 203 (2008).
[20] R. M. Kerr, J. Fluid Mech. **310**, 139 (1996).
[21] R. M. Kerr, Phys. Rev. Lett. **87**, 244502 (2001).
[22] M. van Reeuwijk, H. J. J. Jonker, and K. Hanjalić, Phys. Rev. E **77**, 036311 (2008); **77**, 036312 (2008).
[23] R. Verzicco, Eur. Phys. J. B **35**, 133 (2003).
[24] J. J. Niemela *et al.*, J. Fluid Mech. **449**, 169 (2001).
[25] P. K. Mishra and M. K. Verma, Phys. Rev. E **81**, 056316 (2010).
[26] M. K. Verma, e-print arXiv:1102.251.
[27] D. Lohse and F. Toschi, Phys. Rev. Lett. **90**, 034502 (2003); E. Calzavarini *et al.*, Phys. Fluids **17**, 055107 (2005).
[28] J. P. Whitehead and C. R. Doering, Phys. Rev. Lett. **106**, 244501 (2011); P. Constantin and C. R. Doering, J. Stat. Phys. **94**, 159 (1999).