Energy Spectrum and Flux of Buoyancy-Driven Turbulence

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Gravity or buoyancy plays an important role in atmospheric and geophysical flows. The flow is destabilized when heavier or colder fluid is on top of a lighter or hotter fluid, often seen in thermal convection (Fig 1(a)). Convection plays an important role in interiors of many planets and stars, and it is one of the mechanisms for the generation of a magnetic field. Conversely, the flow is stabilized when a lighter fluid sits on top of a heavier fluid, for example, in Earth's atmosphere (Fig 1(b)). The latter configuration, called stably stratified, can be made turbulent by an additional stirring of the fluid. Kumar et al. [1], and Kumar and Verma [2] examined this problem, and provided a comprehensive theory for the energy spectrum and flux of such flows in a turbulent limit.

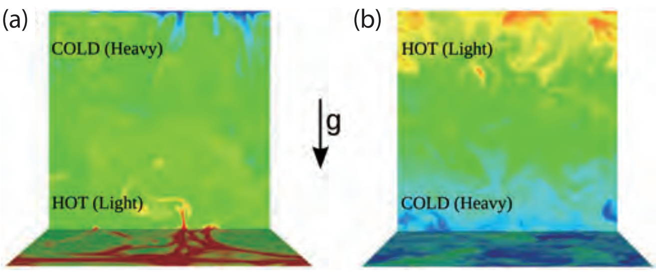

Fig. 1: Density plot of temperature field for (a) Rayleigh-Bénard convection and (b) stably stratified flow.

In a turbulent fluid, for example, in a glass of milk stirred strongly by a spoon, the energy supplied at the large-scale cascades to progressively smaller scales, and finally dissipates to the smallest scale. During this process, a constant energy flux flows across the intermediate length scales, called the inertial range, of the system. However, for stably stratified flows, Bolgiano [3] and Obukhov [4] in 1959 conjectured that buoyancy will convert the kinetic energy to the potential energy at all scales, thus making the kinetic energy flux in the inertial range a decreasing function of wavenumber. This feature makes the physics of buoyancy driven flows quite different from pure fluid turbulence.

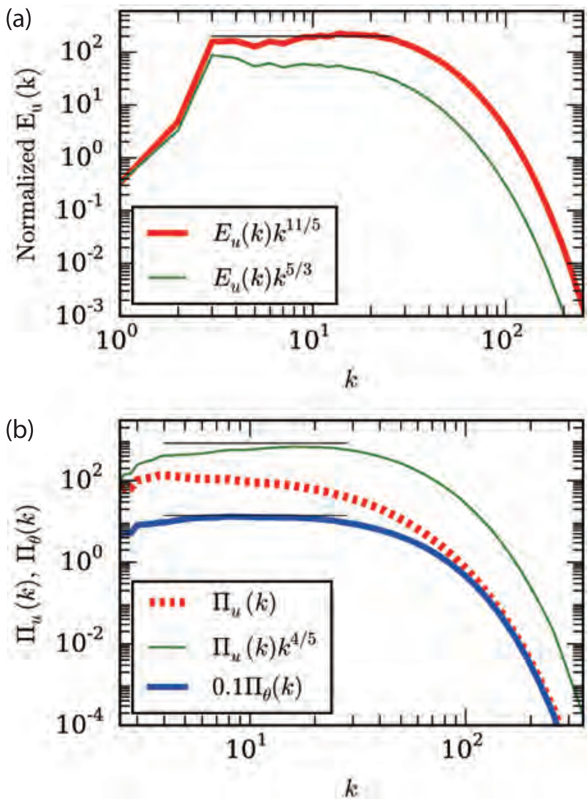

Fig. 2: For stably stratified turbulence plots of (a) normalized kinetic energy spectra $E_u(k)$ with $k^{-11/5}$ and $k^{-5/3}$ and (b) kinetic energy flux $\Pi_u(k)$, normalized KE flux $\Pi_u(k)k^{4/5}$, and potential energy flux $\Pi_\theta(k)$.

Kumar et al. [1] investigated the above aspect of stably-stratified flows using theoretical and numerical tools, and found that Bolgiano and Obukhov's conjecture was indeed correct. They solved the equations of stably stratified flows in the Boussinesq limit using computer simulations. They employed a pseudo-spectral method to solve for the velocity and temperature fields on a 1024^3 grid, and computed the energy spectrum and flux for various parameters. They observed that the flows dominated by buoyancy exhibit $E_u(k) \sim k^{-11/5}$, $\Pi_u(k) \sim k^{-4/5}$, $E_\theta(k) \sim k^{-7/5}$, and $\Pi_\theta(k)$ = constant, where $E_u(k)$ and $E_\theta(k)$ are the energy ($u^2/2$) and entropy ($\theta^2/2$) spectra respectively, and $\Pi_u(k)$ and $\Pi_\theta(k)$ are energy and entropy fluxes respectively. These results, shown in Figs. 2(a) and 2(b), are consistent with the predictions of Bolgiano [3] and Obhukhov [4]. However, for weaker buoyancy, $E_u(k)$ follows Kolmogorov's spectrum with a constant energy flux due to the dominance of the nonlinear term ($u_j \partial_j u_j$). Thus, the simulations of Kumar et al. [1] provide the first numerical validation of Bolgiano and Obukhov's conjecture.

Procaccia and Zeitak [5] and L'vov and Falkovich [6] proposed that the Bolgiano and Obukhov scaling of stably stratified flows would also be applicable to Rayleigh-Bénard convection (RBC). Kumar et al. [1] showed that this is not the case. In RBC, the thermal plumes feed energy to the kinetic energy at all scales. Consequently, for RBC, the kinetic energy flux increases with wavenumber due to buoyancy, not decrease, in contrast to the stably stratified flows. Kumar et al. [1] performed numerical simulations of turbulent RBC for unit Prandtl number, and showed that with wavenumber, the kinetic energy flux increases briefly, and then becomes constant due to a delicate balance of dissipation and energy supply rate. Due to the constancy of energy flux, the turbulent RBC exhibits Kolmogorov's spectrum rather than Bolgiano and Obukhov's $k^{-11/5}$. The spectrum and flux are exhibited in Fig. 3.

Kumar and Verma [2] also constructed a shell model for buoyancy-driven turbulence, and observed similar results as the direct numerical simulations. Moreover, they could achieve higher parameters in the shell model. Kumar and Verma [2] also observed dual scaling ($k^{-11/5}$ followed by $k^{-5/3}$) for stably stratified flows, as proposed by Bolgiano and Obukhov.

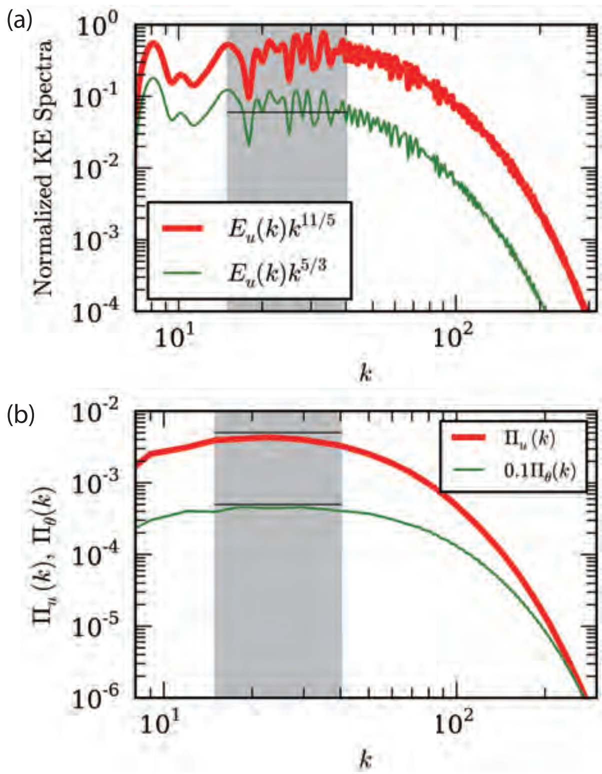

Fig. 3: For RBC simulation (a) plots of normalized KE spectra for Bolgiano-Obukhov scaling and Kolmogorov scaling; (b) KE flux $\Pi_u(k)$ and entropy flux $\Pi_\theta(k)$. The shaded region shows the inertial range.

Thus, Kumar et al. [1] and Kumar and Verma [2] put forward a novel energy flux analysis that deciphers the energy spectrum and flux of buoyancy-driven turbulence. These results would be valuable for modeling the small-scale turbulence in the interiors and atmospheres of planets and stars, as well as in engineering flows.

References

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